Optimal Efficiency as a Design Criterion for Closed Loop Combined Cycle Industrial Cogeneration

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Abstract—In comparison to conventional industrial cogeneration plants, the *closed loop combined cycle cogeneration* (CLCC) system has the advantage that it needs virtually no enhancement in steam handling capability of the existing plant, both in terms of steam pressure as well as volume flow rates. An optimal design procedure is introduced in this paper for configuration of a CLCC system between existing steam headers of an industrial setup.

While aiming primarily at maximization of generation efficiency for the CLCC configuration, the design approach simultaneously attempts to configure a suboptimal number of units for each item of cogeneration equipment. The suboptimal (integer) number of units is maintained within a specified *suboptima margin* of the corresponding optimal (real) number of units that emerges from efficiency maximization.

The design procedure relies heavily on a database including all available makes of cogeneration equipment. Consequently, the approach is very much market dependent, as illustrated adequately by a suitable case study.

Index Terms—Cogeneration, industrial plants, optimization methods.

I. INTRODUCTION

CONVENTIONALLY, steam based industrial cogeneration systems have been set up as one out of three popular configurations [1]-[3], namely topping systems, bottoming systems, and combined cycle systems. A common feature of the three configurations is that their installation within an existing industrial setup usually demands significant enhancement in steam handling capability of the system; the enhancement being in terms of steam pressure levels, volume flow rates, or both [4]. In Section II, this paper describes a variation of the combined cycle concept (referred to as *closed loop combined cycle,* or CLCC for brevity) for industrial plants equipped with at least two steam headers. As distinct from the major capacity enhancements mentioned above, the principal advantage of the CLCC is that its introduction into an existing industrial plant does not warrant such changes. Intended for installation and operation between two of the existing steam headers of the industrial process; the pressure and steam flow requirements of a CLCC plant are strictly within the capability of the concerned headers.

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In industries that have heavy requirement of process steam, such as oil refineries and pulp and paper manufacturing units, existing steam headers differ significantly in terms of pressure as well as steam flow rates [4]. For such industrial plants, the usefulness of a CLCC system may not be restricted by in house requirement of power. As illustrated in subsequent sections of this paper, optimally designed excess cogeneration capability may make surplus power available for sale to consumers or utilities.

Over the years, several attempts to evolve optimal designs for combined cycle power plants have been reported; some of which describe cogeneration applications. In [5], thermoeconomic functionals have been used to design an optimally configured combined cycle system as decided by power requirements. In other works, systems specifically designed for optimality over short and long spans of time have been obtained by single as well as multi objective optimization [6]-[8]. Interaction between optimal configuration design and optimal component design has been effectively employed to achieve overall optimality in [9]. On the other hand, optimization in design of selected components, particularly the air compressor, has been shown to contribute significantly to overall system optimization in [10]. Very recently, system designs that are optimal in the sense of efficiency maximization have been reported in [11], achieving effective generation efficiency in the range of 56-58%. Similar level of generation efficiency appears to be the current trend for most combined cycle cogeneration systems [12], [13].

While most of the above works examine the optimal design problem fundamentally from a thermodynamic viewpoint, the task of a practical system designer is likely to be significantly restricted by the available makes of cogeneration system components (such as gas and steam turbines, and boilers). A design engineer may therefore have to redesign a system configuration entirely subject to availability of components, and this may introduce sub-optimality within an otherwise optimal design to a somewhat uncertain degree. If in addition, the parent industry intends to cogenerate electricity not only for use in plant, but also for sale against economic benefits; then the complexity of a market constrained optimal design may accordingly increase.

As distinct from the works discussed above, this paper describes an *inherently market constrained optimal design procedure* for the CLCC cogeneration system. The configuration design procedure is visualized as a selection problem for optimal "match" between the available makes of steam turbine, HRSG (heat recovery steam generator) boiler, and combustion turbine, so as to maximize the overall combined cycle generation efficiency. It is assumed that other associated components, such as

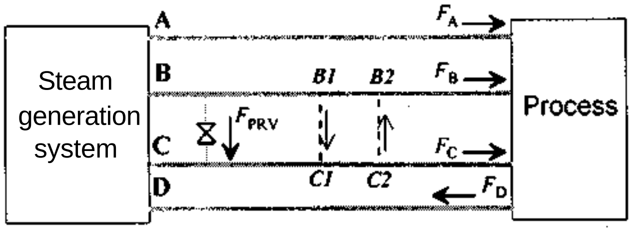

Fig. 1. Steam flow in atypical industrial process employing four headers.

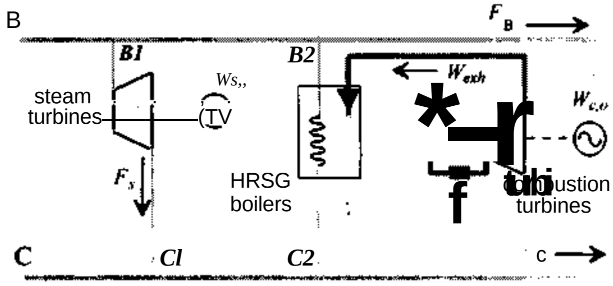

Fig. 2. Generic "closed loop" cogeneration configuration betw een headers B and C of Fig. 1.

alternator for each of the turbines, station transformers, and requisite metering and switchgear are available in compatible ratings. With a knowledge of the present state of art of combined cycle system designs **[11]—[13],** a cycle efficiency of 56% or more is expected.

Section II describes the essential features of the CLCC configuration. Section III introduces the aforementioned market constrained optimal design procedure for a CLCC system, assuming that header data for the parent industry is available. Details of the optimization model are described in Appendices I and II. The twofold objective for selection of each item of cogeneration equipment required by the CLCC setup, is to i) maximize the equipment efficiency of the selected system components, and ii) select the largest possible number of maximal efficiency units, within a permissible maximum number. Simultaneous consideration of both aspects leads to maximization of the overall generation efficiency. As distinct from an earlier algorithm described in [14], this paper uses efficiency maximization is the predominant criterion for configuration.

The nature of practical data required for optimal design of CLCC is described in Section IV. Section V describes an application of the algorithm to a selected case study, indicating the substantial surplus cogenerated power that may be sold against a price, and the economic implications of the optimal design procedure. Section VI presents a discussion on the importance of some critical performance parameters for the design procedure, in view of the formulations of Sections III and IV and the observations of Section V.

II. CLCC COGENERATION BETWEEN EXISTING STEAM HEADERS

The concept of "closed loop" combined cycle is brought out precisely through Figs. 1 and 2. Fig. 1 represents a process with four steam headers, typical of many large industrial establishments such as oil refineries and pulp and paper industries [15], [16]. Each header (-) would typically work at a definite steam pressure *(PA* through P_D, say), with a corresponding steam flow rate *(FA* through F_D, respectively). In certain systems, pressure release valves (PRVs) may exist between specific header pairs; a representative PRV with flow rate F_{PRV} being shown in Fig. 1. In order to configure a CLCC system, all relevant process data (including steam pressure levels and flow rates, as well as exact PRV locations) is assumed to be known *a priori.*

The closed path indicated by a broken line between headers B and C *(B1-C1-C2-B2)* of Fig. 1 is a possible steam path for a CLCC configuration. Assuming that B and C are high and low pressure headers respectively, *Bl* —> Cl indicates a path over which steam enthalpy may be converted to power by installing a set of steam turbines. Likewise C2 —> *B2* indicates a path over which steam may be made to gain enthalpy, such as through a set of appropriately installed HRSGs. Exhaust from a set of combustion turbines may be used as a heat source to the steam generators. The overall cogeneration setup inclusive of the paths *Bl* —> Cl and C2 —> *B2* then constitutes a *closed loop combined cycle* configuration between headers B and C, details of which are shown in Fig. 2. It is easy to visualize alternative CLCC configurations between other header pairs of the same industrial process (say A-B, or A-D, for example).

Regardless of the header pair that is actually selected for CLCC configuration, it is necessary that steam balance for the original process must not be disturbed in any way. For the example of Figs. 1 and 2, this requirement is found to be satisfied if F_s - F_b = FPRV; where steam flow over *Bl* —> Cl is F_s and that over *C2* —> *B2* is *Fb.* The representative PRV shown in Fig. 1 may be thus "absorbed" within the *Bl* —> Cl path.

A CLCC system would output power $W_{Si\,o}$ cogenerated by the steam turbines, and $W_{Ci\,o}$ by the combustion turbines. It will also have two points of input as fuel power, namely, W_{Ci} *fi*, input to the combustion turbine units and W_{by} *fi* (not shown in Fig. 2) input as supplementary fuel to the boilers. Recovered power of an amount W_{exh} is input to the HRSG units from the combustion turbine exhaust.

III. OPTIMIZATION OF THE CLCC CONFIGURATION

In order to design an optimal CLCC configuration for an existing multi-header industrial process, the following issues must be considered:

- Available makes of cogeneration equipment (steam turbines, HRSG boilers, and combustion turbines), to be compiled into appropriate databases as described in Section IV.
- For each item of installed cogeneration equipment, the *maximum suboptima margin* that is permissible between the actual number of units to be installed (suboptimal, integer value) and the number of units obtained as part of the optimization algorithm (optimal, real value).
- Pressure levels of available steam headers.
- Steam flow in each of the available headers.
- Flows through existing PRVs between the headers.

The concept of *suboptima margin* requires some elaboration at this point. If an optimal design indicates that *n* units of a certain equipment are required to be installed (where *n* may not necessarily be an integer), then a practical implementation of the design would involve either ceil[] or floor[] units; where ceil[] is the smallest integer greater than *n* and floor[] is the largest integer less than or equal to *n*. Both ceil[] and floor[] are suboptimal integers close to the optima *n*. Corresponding to each, margins i) (ceil[n] - n)/ceil[n] and ii) (n - floor[n])/floor[n] exists respectively between the optima and suboptima. These margins will be henceforth referred to as the *suboptima margin* for the particular equipment. The *maximum suboptima margins* to be specified for steam turbines *(jj,$_{8t}$)* and gas turbines *(/j,$_{ct}$)* are both defined as ii), since highest integer less than the respective optimal values would result in the maximum possible number of units within optimal steam flow over the closed loop. For HRSG boilers, on the other hand, the maximum suboptima margin *(jiu),* is defined as a maximum admissible margin of type i). This ensures that, despite a minor overdesign in number of units installed, the HRSG units are together able to accommodate the entire steam flow optimized for the steam turbines.

In [14], an optimal design procedure for the CLCC cogeneration system is described that includes consideration of all the above issues except the maximum suboptima margins. In such designs, the optimal choice of steam turbines, boilers, and combustion turbines, may sometimes remain somewhat significantly underutilized. Further, while the optimal choice of steam turbines and combustion turbines is done on the basis of efficiency maximization, the HRSG boilers are selected by matching pressure and flow ratings. The designs obtained by such an approach is therefore strictly not optimal in the sense of maximum efficiency [14].

Appendices I and II describe a new optimal design approach for CLCC configurations that includes all five considerations listed above. An overview of the algorithm is provided as a flowchart in Figs. 3 and 4. The flowchart refers to equation numbers as in Appendix II, thereby indicating the exact usage of the respective equations.

Appendix I provides a detailed list of variables involved in the optimization procedure. Subsections A through H of Appendix II detail various stages of the algorithm itself, essential features of which are mentioned below.

i) The design is sequentially optimized for header pair and steam turbine selections (-) , HRSG boiler selections *(-)* , and combustion turbine selections *(-) ;* in that order (Figs. 3 and 4). Each optimization problem is posed as one of nonlinear programming (NLP) with defined nonlinear objective functions and constraints [17], and may be solved using any of the reliable techniques available to the designer. For the case study described in Section V, the CONOPT constrained optimization solver available as part of the Generalized Algebraic Modeling System (GAMS) language [18] has been used. This essentially employs a computationally efficient form of the well known Generalized Reduced Gradient method (GRG) [19] for NLP problems. The steam flow requirement for steam turbines, evaluated at the end of - is used as a specified data for optimal choice of HRSGs. The heat input requirement for HRSGs, evaluated at the end of - is used as a specified data for optimal choice of combustion turbines.

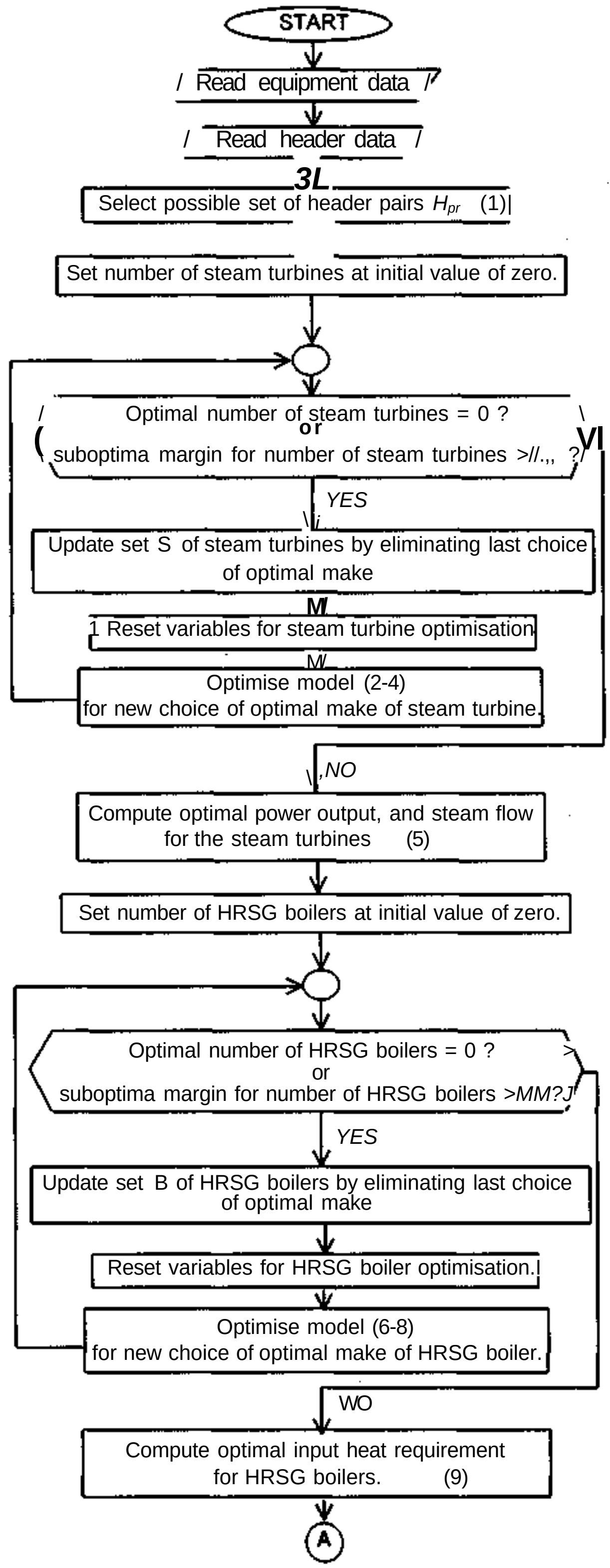

Fig. 3. Flowchart for optimization algorithm (cont'd. to Fig. 4 from A). All equation numbers refer to Appendix II.

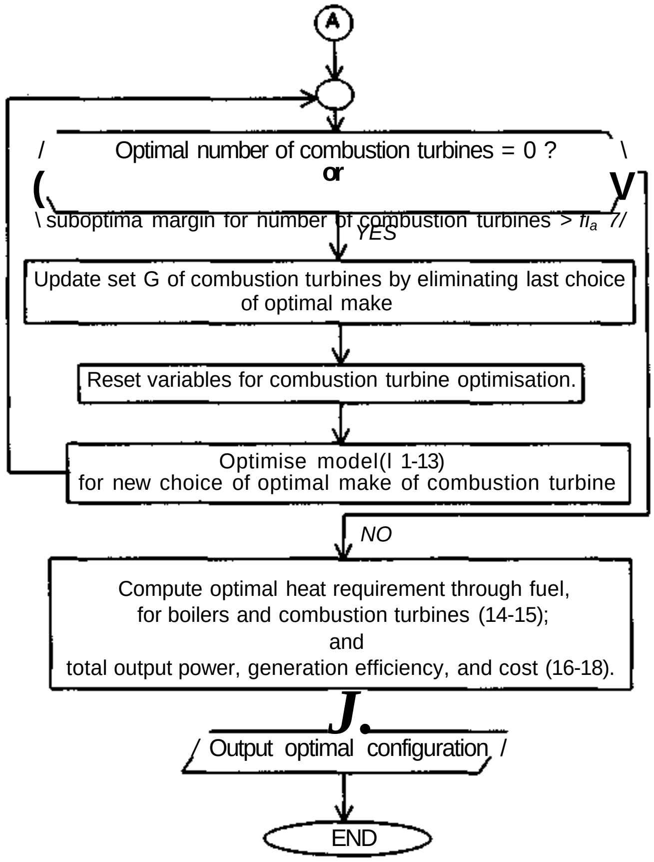

Fig. 4. Flowchart for optimization algorithm (cont'd. after Fig. 3). All equation numbers refer to Appendix II.

ii) Each optimization operates on a known database that includes the available makes of a particular equipment, which is to be compiled out of data available on the makes in question.

iii) Eventually, each stage of optimization described in "i)." above yields a single *optimal make* of the particular equipment from the respective database, and an *optimal number of units* (a real number) of the make to be installed. The objective function of each stage of optimization is the product of the *equipment efficiency* and *number of units*, which is maximized over the respective database of available makes. The optimal make is indicated by setting a *selection flag* to unity while other similar flags are set to zero.

iv) Following execution of the optimization algorithm for a particular equipment as described in "iii)" above, the suboptima margin for each installed unit is computed for the suboptimal (integer) number of units close to the optima. If the margin is more than the respective maximum value (Mst, *IJ'bi, /Jet)*, then the choice of the equipment *make* and *number of units* fail to meet the expected suboptima margin goal (Figs. 3 and 4). The database of available makes is then reduced by eliminating the particular make, and the optimization is again executed on the smaller database. This process is continued until the suboptima margin for an optimal make is less than the respective */J,* (Sections C, E, and G of Appendix II).

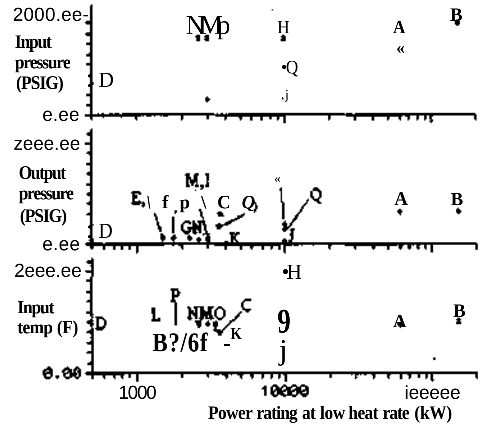

Fig. 5. Overview of data on steam turbines (17 makes).

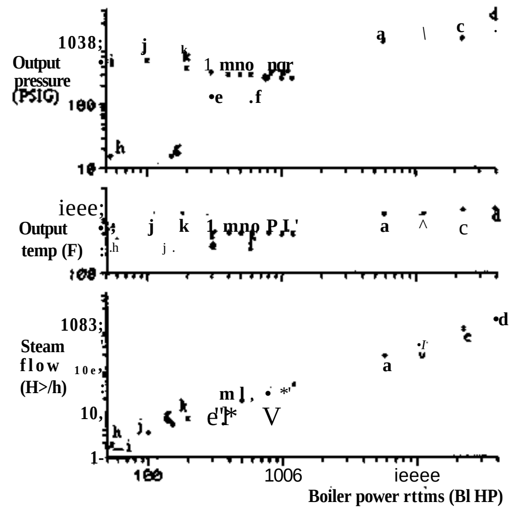

Fig. 6. Overview of data on HRSG boilers (18 makes).

To conclude the optimization process, a) the total output power, b) the overall generation efficiency, and c) the total equipment cost (involving steam turbines, boilers, and combustion turbines), can be computed using the data available for the different makes [(16)-(18) of Appendix II].

IV DATABASE FOR COGENERATION EQUIPMENT

In practice, the available makes, of steam turbine, HRSG boiler, and combustion turbine would constitute the *market* for the three major items of cogeneration equipment; and these may vary considerably between different cases to which the algorithm is applied.

For illustrative purposes, particularly with reference to the case study presented in the next section, data on seventeen makes of steam turbine, eighteen makes of HRSG boiler, and twenty five makes of combustion turbine have been compiled to represent *markets* of available makes. Detailed specification sheets included in [16] for different makes of cogeneration equipment have been used for this compilation. Overview of the resulting database on steam turbines (makes A-Q), HRSG boilers (makes a-r) and combustion turbines (makes 1-25), are shown in Figs. 5-7. In each figure, important specifications

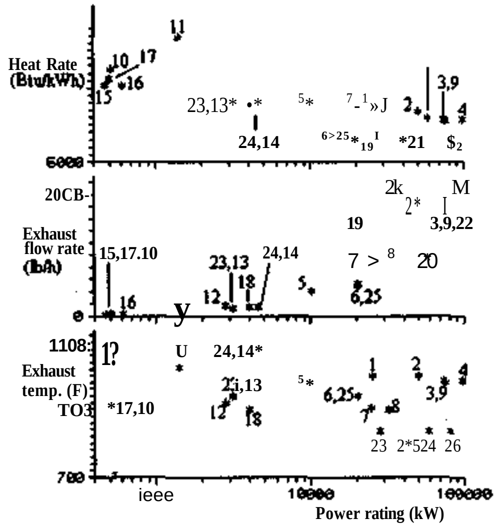

Fig. 7. Overview of data on combustion turbines (25 makes).

are plotted against the power rating for different makes of cogeneration equipment. An exhaustive list of the required specifications is provided below (all cost data in any appropriate currency unit).

Data on Steam Turbines:

Specifications Obtained from Datasheets: **Rating for high** and low heat rates (kW), inlet and outlet pressures (PSIG), inlet temperature (F), cost of turbine unit.

Other Quantities Computed from the Specifications, Using Closed Form Thermodynamic Functions [20], [21]: **Inlet enthalpy** (kJ/kg), outlet enthalpy (kJ/kg), turbine efficiency for high and low heat rates.

Data on HRSG Boilers:

Specifications Obtained from Datasheets: **Power rating** (B1-HP), output steam pressure (PSIG), output steam temperature (F), output steam flow rate (lb/h), cost of boiler unit.

Other Quantities Computed from the Specifications, Using Closed Form Thermodynamic Functions [21]: **Output steam** enthalpy (kJ/kg), and boiler efficiency.

Data on Combustion Turbines:

Specifications Obtained from Datasheets: **Power rating** (kW), heat rate (Btu/kWh), and exhaust and ambient temperatures (F), cost of combustion turbine unit.

Other Quantities Computed from the Specifications, Using Closed Form Thermodynamic Functions [21]: Exhaust **and** ambient enthalpy (kJ/kg), and exhaust power (kW).

V. CASE STUDY

The process data shown in Table I refers to the steam header system of a pulp and paper industry, adapted from a detailed industrial setup described in [16]. This industry uses four headers, of which temperature of steam in the 575 psig header is known to be 740F. The temperatures for the remaining headers can be obtained using the known closed form expressions for adiabatic steam expansion, that are documented in [21]. The industry has a requirement of electric power amounting to 40 MW, which is purchased from the local utility by the parent plant [16].

TABLE I
HEADER DATA FOR CASE STUDY

Header	Pressure (psig)	Steam flow (lb/h)
H1	575	1 364 000
H2	300	141 000
H3	130	172 000
H4	50	790 000
Pressure release valve locations		PRV flows (lb/h)
H1→H2		30 000
H2→H3		25 000
H3→H4		23 000

For different values of *maximum number of units permissible* for steam turbines (N_{st}^{max}), HRSG boilers (N_{hb}^{max}), and combustion turbines (N_{ct}^{max}), the optimal configurations obtained are as detailed in Table II. It is observed that with a small permissible number of units ($N_{st}^{max} = N_{hb}^{max} = N_{ct}^{max} = 1, 2$) the optimal CLCC configurations are obtained between headers H2 and H4; while with larger number of units, the optima invariably settles for the H1-H4 pair. A reference to Table I indicates that this is to be expected since the steam flow available in H1 is substantially greater than that available in H2. Thus any CLCC that requires large enthalpy changes is more likely to converge to an optima involving the H1-H4 pair in preference to others.

Each *market* database typically consists of discrete makes, so that the desired low values of suboptima margin may not always be possible. However, it is observed that optimization for steam turbines leads to negligibly low suboptima margin in most cases, because these are designed in terms of *number of units* as well as *unit rating* (refer A-C of Appendix II), which leads to excellent unit sizing. The same is not the case with the remaining items of equipment, for which makes are available at discrete ratings, and the optimization essentially decides only the *number of units* to be installed.

Low suboptima margins are particularly rare when the *maximum number of units* permitted is small. In Table II, when $N_{st}^{max} = N_{ct}^{max} = N^{max} = 1$, a [illegible] mismatch is obtained for the boilers (i.e., roughly 45% of the installed boiler capacity is expected to remain unutilized). Further, the heat requirement of the boilers is too low for any combustion turbine to be included in the CLCC configuration, so that the boilers must be directly fired.

The design improves somewhat when at most two units of each item of equipment is permitted. While the suboptima margin for boilers is still quite high, both the output power as well as the cost are found to improve, coupled with a high generation efficiency. For the case study in question, the best configuration is evolved when at most three units of each item of equipment is permitted. This immediately requires three units of optimally selected make of steam turbine, two of combustion turbine, and one of boiler; the suboptima margin being less than 4.0% for all equipment. Due to the good match

TABLE II
OPTIMAL CLCC CONFIGURATIONS FOR PULP AND PAPER PLANT

MF™= N_N^{max} = MV" = 1		
Optimal header pair:		H2, H4
Optimal configuration:		
Steam turbines:	make J, 1 unit	*(suboptima margin: zero)*
HRSG boilers:	make c, 1 unit	*(suboptima margin: 0.449)*
Combustion turbines:	nil	
Total generation capacity:		10.00 MW
Generation efficiency:		63.4%
Cost of equipment:		$ 322/- per kW
$N,r^{ax} = N_{bl}^{max} = N_{ct}^{max*} = 2$		
Optimal header pair:		H2.H4
Optimal configuration:		
Steam turbines:	make J, 2 units	*(suboptima margin: zero)*
HRSG boilers:	make d, 1 unit	*(suboptima margin: 0.357)*
Combustion turbines:	make 6, 1 unit	*(suboptima margin: 0.001)*
Total generation capacity:		40.10 MW
Generation efficiency:		69.1%
Cost of equipment:		$ 225/- per kW
$N_{st}^{max} = N_{bl}^{max} = N_{ct}^{max} = 3$		
Optimal header pair:		HI, H4
Optimal configuration:		
Steam turbines:	make J, 3 units	*(suboptima margin: zero)*
HRSG boilers:	make d, 1 unit	*(suboptima margin: 0.035)*
Combustion turbines:	make 25,2 units	*(suboptima margin: 0.037)*
Total generation capacity:		70.20 MW
Generation efficiency:		59.5%
Cost of equipment:		$ 179/- per kW
Mi$^{1™"1}$ = M/$^{™"1}$ = Mi$^{1™"}$ = 4		
Optimal header pair:		H1.H4
Optimal configuration:		
Steam turbines:	make J, 4 units	*(suboptima margin: zero)*
HRSG boilers:	make c, 3 units	*(suboptima margin:* ft. *265)*
Combustion turbines:	make 6, 3 units	*(suboptima margin: 0.005)*
Total generation capacity:		100.30 MW
Generation efficiency:		57.8%
Cost of equipment:		$200/-per kW
Mi"^{ax} = N_{bl}^{max} = Mr™ = 5		
Optimal header pair:		H1.H4
Optimal configuration:		
Steam turbines:	make *i*, 4 units	*(suboptima margin: 0.205)*
HRSG boilers:	make b, 5 units	*(suboptima margin: 0.164)*
Combustion turbines'.	make 5,3 units	*(suboptima margin: 0.068)*
Total generation capacity:		70.60 MW
Generation efficiency:		55.6%
Cost of equipment:		$ 203/- per kW

between the equipment, the cost of installed capacity is seen to drop to $179/-per kilowatt, with an efficiency of 59.5%. The CLCC would now generate more than three times the power requirement of the host plant, leaving a substantial amount for sale. The generation efficiency, though somewhat lower than in the previous case, is well above the current trend for combined cycle plants [11]—[13].

$jymax_{st}$ = $jymax_{bl}$ = $jymax_{ct}$ = 2 or 3 appear to lead to acceptable configuration of CLCC for the plant in question. The tradeoff between the two is evidently one of cost against efficiency. It is further observed from Table II that increasing the permissible number of units beyond three leads to configurations that are both expensive as well as poor in efficiency. It follows that for a particular industrial plant, there is a certain small number of CLCC equipment for which the overall equipment cost is minimal, while the overall generation efficiency too assumes acceptable values. This fact is observed in other similar case studies as well.

VI. CRITICAL DETERMINANTS OF THE OPTIMA

The optimization problem described in Section III (with details included in Appendix II) is far from being a simple single stage nonlinear programming problem. The sequential optimization process would expectedly lead to optima that could well differ from those obtained by a simple single stage NLP formulation. Specifically, the sequential behavior of the present algorithm can be viewed in the following aspects:

- The optimal choice of combustion turbine units follows the optimal choice of HRSG units, which in turn follows the optimal choice of steam turbine units. The optima obtained for each stage depends significantly on those of the previous stages.
- Within each of the three stages, the hyperspace for optimization is reduced by one make in every iterative loop. As a particular make is excluded, the variables related to it are also eliminated from the subsequent iterations of optimization. (For example, in each iteration for the optimization of steam turbines, one S_{st}, variable, one N_{st}, variable, one R_{st}, variable, and one rj_{st}, variable are simultaneously excluded.) Each iteration therefore involves an optimization within a hyperspace of progressively lower dimension.

Clearly these two features would make the final optima differ significantly from those obtained through any single stage model.

Two factors play a decisive role in determining the sequential progress of the optimization process, and it is important to understand the impact that these have on the overall algorithm.

- The maximum permissible number of units, designated $7V^{max}$ for each item of cogeneration equipment, which offers an important constraint to each sequentially restricted iteration described above.
- The maximum suboptima margin, designated */J,* for each item of cogeneration equipment, which decides whether a) the optimization should proceed through a subsequent iteration or not, and b) the make that needs to be eliminated in order to generate a reduced hyperspace for the subsequent iteration.

It is quite difficult to set precise guidelines for selection of either $7V^{max}$ or *fj,* for any of the three items of cogeneration equipment. Generally, each $7V^{max}$ should be a small integer as close to unity as possible; since such a choice would lead to overall a small number of units to be installed. Likewise, each */J,* parameter should be as close to zero as possible so as to have close match between the required capacity and that of the selected equipment. However, more specific selection norms for either parameter is not easy to state; and one must simply attempt to make these as small as possible.

The reason for this unavoidable arbitrariness is easy to appreciate in view of the fact that the optimal configuration of CLCC system, as described in this paper, is not only *process dependent*, but also very much *market dependent.* Consider for instance, the optimal selection of combustion turbines for a particular industrial process; following the selection of steam turbines and HRSGs. The requirement of exhaust temperature and enthalpy may be such that out of the available range, the optimal make may require two units to be installed with a resulting suboptima margin of 6.0%. It follows that an optima can be realized only if the value of -/V"$_{ct}^{iax}$ is set to be 2.0 or higher; while the value of *fid,* must be set to a value of 0.06 or higher. However, addition of a new make to the set G (thereby expanding upon the available range), which may happen to be better matched to the exhaust requirements for the process in question, may lead to an optimal choice that requires only one unit of combustion turbine with a suboptima margin of only (say) 2.0%! It follows that with the availability of the new make in question, N_{ct}^{max} can be reduced to 1.0 and /u$_{ct}$ to 0.02.

Finally, it may be noted that certain simple variations can be attempted over the optimal CLCC configuration algorithm, so as to yield better results in specific cases. One obvious modification would be to allow different makes of any piece of cogeneration equipment to be simultaneously selected (two different makes of steam turbines, for example), and this may sometimes lead to higher generation efficiencies and lower cost of installed capacity. However, such modifications do not significantly augment the basic concepts outlined in Section III and Appendix II, and may be simply incorporated as and when appropriate.

VII. CONCLUSION

This paper has further substantiated the idea brought forth in an earlier work by the author [14], that significant industrial cogeneration of power is possible *between existing steam headers* of a large industrial establishment by suitably configured closed loop combined cycle (CLCC) cogeneration systems.

The optimal configuration of the CLCC cogeneration system is heavily dependent on the available makes of cogeneration equipment, which may be taken to constitute a *market.* This feature makes the design approach presented above more realistic than otherwise.

APPENDIX I
DEFINITION OF VARIABLES AND DATA

Header Definitions:

H: Set of all available steam headers in the industry.

h: Generic index for each header available *h* e H.

P(h), T(h), F(h), H(h): Pressure (MPa), temperature (K), flow rate (kg/s), and enthalpy (kJ/kg) of header *heU.*

Up,.: Set of header pairs *(hi, hj)* between which adiabatic processes may be set up in order to generate electric power, where *hi, hj* e H *and P(hi) > P(hj).*

Fpnv(hi, hj): Steam flow (kg/s) through a pressure release valve between headers *hi* and *hj.*

Steam Turbine Definitions:

S: Set of makes available, and used to define the market.

s: Generic index for makes available, ***s e S.***

R$_s$t,hh(s), R$_s$t,ih(s), Vst,hh(s), Vst,ih(s), Pst,in(s), P$_s$t,out(s), H$_s$t, in(«), *H$_{st>}$* ou t *(s), rjst(s), c$_{st}$(s):* Rating (kW) for high heat rate, rating (kW) for low heat rate, efficiency for high and low heat rates, inlet pressure (MPa), outlet pressure (MPa), inlet enthalpy (kJ/kg), outlet enthalpy (kJ/kg), turbine efficiency, and unit cost; all for make *s* e S.

S$_{st}$(hi, hj, s): Selection flag set to unity if make s £ Sis chosen to operate between *(hi, hj)* e H$_{pr}$, zero otherwise.

N$_{st}$(hi, hj, s), F$_{st}$(hi, hj, s), R$_{Bt}$(hi, hj, s), W$_s$(hi, hj, s): Number of units, steam flow (kg/s) in each unit, rating (kW) of a unit, total output power (kW) from all units; all with reference to make *s* e S operating between *(hi, hj)* e H$_{pr}$.

W$_{SiO}$, F$_s$: Maximum power output (kW), and maximum steam flow (kg/s) expected of the complete steam turbine configuration.

lj,$_{st}$: Maximum suboptima margin for steam turbine units.

7V"$_{st}^{iax}$: Maximum number of steam turbine units permissible.

HRSG Boiler Definitions:

B: Set of makes available, and used to define the market.

b: Generic index for makes available, *b* e B.

Rbi(b), Pbi(b), Tbi(b), Fbi(b), Hbi(b), rjbi(b), Cbi (b): Power rating (kW), output steam pressure (MPa), output steam temperature (K), output steam flow rate (kg/s), output steam enthalpy (kJ/kg), boiler efficiency, and unit cost; all for make *b* e B.

Sbi(hi, hj,b): Selection flag set to unity if make *b e* B is chosen to operate between *(hi, hj)* e *Hpr,* zero otherwise.

Nbi(hi, hj, b): Number of boiler units, with reference to make *b* e B operating between *(hi, hj)* e Hj,r.

W$_{bi}$i,W$_{bi}$fi: Total input power required by boilers (kW), and input power required (kW) as supplementary fuel heat.

F$_b$: Total steam flow (kg/s) through the HRSG units.

fibf. Maximum suboptima margin for HRSG boilers.

7Vjjy^{ax}: Maximum number of HRSG boiler units permissible.

Combustion Turbine Definitions:

G:	Set of makes available, and used to define the market.
g:	Generic index for makes available, $j \in G$.
$R_{ct}(g), P_{ct}(g)$, $T_{ct,exh}(g)$, $T_{ct,amb}(g)$, $H_{ct,exh}(g)$, $H_{ct,amb}(g)$, $F_{ct,exh}(g)$, $W_{ct,exh}(g)$, $\eta_{ct}(g)$, $c_{ct}(g)$:	Power rating (kW), heat rate, and exhaust and ambient temperatures (K), exhaust and ambient enthalpy (kJ/kg), exhaust power (kW), combustion turbine efficiency, and unit cost; all for make $g \in G$.
$S_{ct}(hi, hj, g)$:	Selection flag set to unity if make $g \in G$ is chosen to supply heat to a boiler operating between $(hi, hj) \in H_{pr}$, zero otherwise.
$N_{ct}(h_i, h_j, g)$, $W_c(hi, hj, g)$:	Number of units, total output power (kW); all with reference to make $g \in G$ that supplies exhaust heat to a boiler operating between $(hi, hj) \in H_{pr}$.
$W_{c,o}, W_{c,fi}$:	Maximum power output (kW), and maximum total input power (kW) required as fuel.
μ_{ct}:	Maximum suboptima margin for combustion turbine units.
N_{ct}^{max}:	Maximum number of combustion turbine units permissible.

APPENDIX II
OPTIMIZATION OF "CLOSED LOOP" COGENERATION

In the formulation to follow, asterisked variables indicate optimal values.

The overall optimization algorithm has already been summarized in Figs. 3 and 4. Details of the concerned equations are provided in this Appendix.

A. Identification of Header Pairs

Out of the set H of available steam headers, pairs (hi, hj) between which expansion of steam is possible are identified, and a subset H_{pr} is defined to contain these.

$$\{(hi, hj) \in H_{pr} \text{ if } h_i, hj \in H, \wedge P(h_i) > P(hj). \tag{1}$$

B. Optimization for Steam Turbines

The objective is to maximize a product of a) the number $N_{st}(hi, hj, s)$, and b) unit efficiency $\eta_{st}(s)$, over all possible makes $(s \in S)$ that may be made to operate between the best choice of header pair $(hi, hj) \in H_{pr}$. (The optima is indicated by flag $S_{st}(hi, hj, s)$ set to 1.)

$$E_s^* \triangleq \max_{\substack{s \in S_{ij} \\ (h_i, h_j) \in H_{pr}}} \{E_s(hi, hj, s)\}; \tag{2}$$

$$S_{ij} \triangleq \{\forall s \in S: P_{st,in}(s) < P(hi) \wedge P_{st,out}(s) > P(hj), (hi, hj) \in H_{pr}\}$$

where

$$E_s(hi, hj, s) \triangleq S_{st}(hi, hj, s) \cdot N_{st}(hi, hj, s) \cdot \eta_{st}(s). \tag{3}$$

The constraints for this optimization are defined as (4). Except the first equation, the rest are defined individually for each $(hi, hj) \in H_{pr}$ and each $s \in S_{ij}$,

$$\sum_{\substack{s \in S_{ij} \\ (h_i, h_j) \in H_{pr}}} S_{st}(hi, hj, s) = 1; \quad 0 \leqslant S_{st}(hi, hj, s) \leqslant 1;$$

$$F_{st}(hi, hj, s) \cdot \eta_{st}(s) \cdot [H_{st,in}(s) - H_{st,out}(s)] = R_{st}(hi, hj, s);$$

$$F_{PRV}(hi, hj) \leqslant N_{st}(hi, hj, s) \cdot F_{st}(hi, hj, s) \leqslant \max\{F(hi), F(hj)\};$$

$$R_{st,lh}(s) \leqslant R_{st}(hi, hj, s) \leqslant R_{st,hh}(s);$$

$$\eta_{st,lh}(s) \leqslant \eta_{st}(s) \leqslant \eta_{st,hh}(s);$$

$$N_{st}(hi, hj, s) \leqslant N_{st}^{max}. \tag{4}$$

In the above, $R_{st,lh}(s)$ and $R_{st,hh}(s)$ are specified turbine ratings corresponding to low and high heat rates for the particular make $s \in S$, while $\eta_{st,lh}(s)$ and $\eta_{st,hh}(s)$ are the corresponding efficiency values, computed *a priori* from de Laval curves [20]. The values for inlet and outlet steam enthalpy $[H_{st,in}(s)$ and $H_{st,out}(s)$, used in the third constraint of (4)] can also be computed *a priori* as thermodynamic closed form functions of the respective steam temperature and pressure $[P_{st,in}(s)$ and $P_{st,out}(s)]$, as documented in [21].

C. Execution of Model (2)—(4) as Shown in Figs. 3 and 4

The solution is an *optimal make* $s^* \in S$ operating between the *optimal header pair* $(h_i^*, h_j^*) \in H_{pr}$ with a suboptima margin that is not more than μ_{st}. Corresponding to this, the *maximum power output* from the steam turbine units, and the *maximum steam flow* through them, are computed as

$$W_{lo} = \text{floor}[N_{st}^*(h_i^*, h_j^*, s^*)] \cdot R_{st}(h_i^*, h_j^*, s^*)$$

$$F_s^* = \text{floor}[N_{st}^*(h_i^*, h_j^*, s^*)] \cdot F_{st}(h_i^*, h_j^*, s^*). \tag{5}$$

D. Optimization for HRSG Boilers

The objective is to maximize a product of a) the number $N_{bl}(hi, hj, b)$, and b) unit efficiency $\eta_{bl}(b)$, over all possible makes $(b \in B)$ that may be made to operate between the best choice of header pair $(hi, hj) \in H_{pr}$ (already optimized as described in B-C). (The optima is indicated by flag $S_{bl}(hi, hj, b)$ set to 1.)

$$E_b^* \triangleq \max_{b \in B_{ij}} \{E_b(h_i^*, h_j^*, b)\}; \tag{6}$$

$$B_{ij} \triangleq \{\forall b \in B: P_{bl}(b) > P(h_i^*) \wedge T_{bl}(b) > T(h_i^*), (h_i^*, h_j^*) \in H_{pr}\}$$

where

$$E_b(h_i^*, h_j^*, b) \triangleq S_{bl}(h_i^*, h_j^*, b) \cdot N_{bl}(h_i^*, h_j^*, b) \cdot \eta_{bl}(h_i^*, h_j^*, b). \quad (7)$$

The constraints for this optimization are defined as (8). Except the first equation, the rest are defined individually for each $b \in B_{ij}$,

$$\sum_{b \in B_{ij}} S_{bl}(h_i^*, h_j^*, b) = 1; \quad 0 \leq S_{bl}(h_i^*, h_j^*, b) \leq 1 \quad (8)$$

$$\sum_{b \in B_{ij}} S_{bl}(h_i^*, h_j^*, b) N_{bl}(h_i^*, h_j^*, b) F_{bl}(b) = F_i^* - F_{Pnv}(h_i^*, h_j^*)$$

$$N_{bl}(h_i^*, h_j^*, b) \leqslant N_{bl}^{\max}.$$

E. Execution of Model (6)—(8) for as Shown in Figs. 3 and 4

The solution is an *optimal make* $b^* \in B$ operating between the *optimal header pair* $(h_i^*, h_j^*) \in H_{pr}$ with a maximum suboptima margin μ_b. The *total input heat requirement* of the boilers may be computed in kW as

$$W_{b,i}^* = F_s^* \cdot [H(h_i^*) - H(h_j^*)]/\eta_{bl}(b^*). \quad (9)$$

$\eta_{bl}(b)$ may in certain cases be specified for all $b \in B$. Alternately, $\eta_{bl}(b^*)$ can be computed from the outlet enthalpy of steam $H_{bl}(b^*)$, steam flow rate $F_{bl}(b^*)$, and power rating $R_{bl}(b^*)$ as

$$\eta_{bl}(b^*) = R_{bl}(b^*)/[H_{bl}(b^*) \cdot F_{bl}(b^*)]. \quad (10)$$

The enthalpy level for the headers $[H(h_i^*)$ and $H(h_j^*)]$, as well as the boiler outlet $(H_{bl}(b^*))$ are easy to compute as closed form functions of the respective temperatures and pressures [21].

F. Optimization for Combustion Turbines

The objective is to maximize a product of a) the number $N_{ct}(h_i, h_j, g)$, and b) unit efficiency $\eta_{ct}(g)$, over all possible makes $(g \in G)$ that may be made to operate between the best choice of header pair $(h_i, h_j) \in H_{pr}$ (already optimized as described in B-C). (The optima is indicated by flag $S_{ct}(h_i, h_j, g)$ set to 1.)

$$E_c^* \triangleq \max_{g \in G_{ij}} \{E_c(h_i^*, h_j^*, g)\} \quad (11)$$

$$G_{ij} \triangleq \{\forall_g \in G: T_{ct,exh}(g) \geq T(h_i^*), (h_i^*, h_j^*) \in H_{pr}\}$$

where

$$E_c(h_i^*, h_j^*, g) \triangleq S_{ct}(h_i^*, h_j^*, g) \cdot N_{ct}(h_i^*, h_j^*, g) \cdot \eta_{ct}(g). \quad (12)$$

The constraints for this optimization are defined as (13). Except the first equation, the rest are defined individually for each $g \in G_{ij}$,

$$\sum_{g \in G_{ij}} S_{ct}(h_i^*, h_j^*, g) = 1; \quad 0 \leq S_{ct}(h_i^*, h_j^*, g) \leq 1; \quad (13)$$

$$\sum_{g \in G_{ij}} S_{ct}(h_i^*, h_j^*, g) \cdot N_{ct}(h_i^*, h_j^*, g) \cdot W_{ct,exh}(g) = W_i^*$$

$$N_{ct}(h_i, h_j, g) \leq N_{ct}^{\max}.$$

For any $g \in G$, the heat content of exhaust (kW) can be computed *a priori* as

$$W_{ct,exh}(g) = F_{ct,exh}(g) \cdot [H_{ct,exh}(g) - H_{ct,amb}(g)]. \quad (14)$$

The exhaust flow rate $F_{ct,exh}(g)$ is usually a specified data for any make of combustion turbine. The enthalpy levels for turbine exhaust as well as the turbine ambient $[H_{ct,exh}(g)$ and $H_{ct,amb}(g)$, respectively] may be computed as closed form polynomial functions of the respective temperatures $[T_{ct,exh}(g)$ and $T_{ct,amb}(g)$, which are specified] [21].

G. Execution of Model (11)—(13) as Shown in Figs. 3 and 4.

The solution is an *optimal make* $g^* \in G$ operating with the already selected makes of steam turbine and HRSG boiler $(s^* \in S, b^* \in B)$, with maximum suboptima margin of μ_{ct}. The power output of the combustion turbines, their fuel heat (kW) requirement, and the supplementary fuel heat (kW) requirement of the HRSG boilers may be computed respectively as

$$W_{c,o}^* = \text{floor}[N_{ct}(h_i^*, h_j^*, g^*)] \cdot R_{ct}(g^*)$$

$$W_{c,fl}^* = W_{c,o}^* \cdot \rho_{ct}(g^*) \quad (15)$$

$$W_{b,fl}^* = W_{b,i}^* - \text{floor}[N_{ct}(h_i^*, h_j^*, g^*)] \cdot W_{ct,exh}(g^*).$$

H. Partial Assessment of Cogeneration System

The total requirement of heat input (kW) can be easily computed from (16) as

$$W_{fl}^* \triangleq W_{c,fl}^* + W_{b,fl}^*. \quad (16)$$

Most boilers and combustion turbines that are designed specifically for cogeneration systems, can operate on a variety of fuels [16]. Thus the choice of fuel is essentially one of *heat value per unit mass,* subject to availability. Once a particular fuel is chosen, the running fuel cost of the cogeneration unit can be evaluated from the input heat requirement obtained as (16).

The total output power is given by

$$W_o^* = W_{c,o}^* + W_{s,o}^* \quad (17)$$

and this may be used to evaluate the effective thermal efficiency of the optimal configuration as $\eta_{cogen} \triangleq (W_o^*/W_{fl}^*)$.

The total cost of installed capacity for the three items of equipment can be evaluated as

$$Cost = \{\text{floor}[N_{st}(h_i^*, h_j^*, s^*)] \cdot c_{st}(s^*)$$
$$+ \text{ceil}[N_{st}^*(h_i^*, h_j^*, s^*)] \cdot c_{bl}(b^*)$$
$$+ \text{floor}[N_{ct}(h_i^*, h_j^*, g^*)] \cdot c_{ct}(g^*)\}. \quad (18)$$

REFERENCES

[1] H. L. Harkins, "PURPA: New horizons for electric power utilities and industry," *IEEE Trans, on Power Apparatus and Systems,* vol. PAS-100, no. 6, pp. 2784-2789, 1981.
[2] D. H. Brown, "Cogeneration potential of energy conversion systems," *IEEE Trans, on Power Apparatus and Systems,* vol. PAS-101, no. 8, pp. 2597-2601, 1982.
[3] B. W. Wilkinson and R. W. Barnes, *Cogeneration of Electricity and Useful Heat:* CRC Press Inc., 1980, pp. 17-62.
[4] K. A. Kirkby, J. F. Rich, and P. J. Mahoney, "Cogeneration systems evaluation: A case study," *IEEE Trans, on Power Apparatus and Systems,* vol. PAS-100, no. 6, pp. 2790-2795, 1981.
[5] C. A. Frangopoulos, "Optimal synthesis and operation of thermal systems by the thermoeconomic functional approach," *Trans. of ASME: Journal of Engineering for Gas Turbines and Power,* vol. 114, no. 4, pp. 707-714, 1992.
[6] R. Yokoyama and K. Ito, "Multi objective optimization in unit sizing of a gas turbine cogeneration plant," *Trans. of ASME: Journal of Engineering for Gas Turbines and Power,* vol. 117, no. 1, pp. 53-59, 1995.
[7] K. Ito, R. Yokoyama, and Y Matsumoto, "Optimal operation of cogeneration plants with steam injected gas turbines," *Trans. of ASME: Journal of Engineering for Gas Turbines and Power,* vol. 117, no. 1, pp. 60-66, 1995.

[8] R. Yokoyama, K. Ito, and Y. Matsumoto, "Optimal multistage expansion planning of a gas turbine cogeneration plant," *Trans. ofASME: Journal of Engineering for Gas Turbines and Power,* **vol. 118, no.** 4, pp. 803-809, 1996.

[9] Y. M. El-Sayed, "A second-law-based optimization: Parts 1 (methodology) **and 2 (application),"** *Trans. ofASME: Journal of Engineering for Gas Turbines and Power,* vol. 118, no. 4, pp. 693-703, 1996.

[10] F. Bozza, A. Senatore, and R. Tuccillo, "Thermal cycle analysis for component aerodesign for gas turbine concept in low-range cogenerating systems," *Trans. ofASME: Journal of Engineering for Gas Turbines and Power,* vol. 118, no. 4, pp. 792-802, 1996.

[11] P. J. Dechamps, "Advanced combined cycle alternatives with latest gas **turbines,"** *Trans. ofASME: Journal of Engineering for Gas Turbines and Power,* vol. 120, no. 2, pp. 350-357, 1998.

[12] O. Bolland and J. F. Stadaas, "Comparative evaluation of combined cycles and gas turbine systems with water injection, steam injection, and **recuperation,"** *Trans. ofASME: Journal of Engineering for Gas Turbines and Power,* vol. 117, no. 1, pp. 138-145, 1995.

[13] J. M. Beér and R. V. Garland, "A coal-fueled combustion turbine cogeneration system with topping combustion," *Trans. ofASME: Journal of Engineering for Gas Turbines and Power,* **vol. 119, no. 1, pp. 84-92,** 1997.

[14] S. Roy, "An approach to configure optimal cogeneration system between existing steam headers of an industry," *International Journal of Energy Research,* vol. 23, no. 7, pp. 625-635, 1999.

[15] T. G. Hicks, *Power Plant Evaluation and Design Reference Guide:* **Mc-**Graw Hill Book Company, 1986, sec. 1-4.

[16] S. A. **Spiewak,** *Cogeneration and Small Power Production Manual:* **The** Fairmont Press Inc., 1987.

[17] A. Ravindran, D. T. Phillips, and J. J. Solberg, *Operations Research: Principles and Practice:* **John Wiley & Sons, 1987.**

[18] A. S. Drud, "GAMS/CONOPT," in *GAMS—The Solver Manuals:* GAMS Development Corporation, 1993.

[19] J. Abadie and J. Carpentier, "Generalization of the Wolfe reduced gradient method to the case of nonlinear constraints," in *Optimization,* R. Fletcher, Ed: Academic Press, 1969, pp. 37-47.

[20] *Transamerica Delaval Engineering Handbook,* **McGraw Hill Book** Company, 1983, pp. 5-11.

[21] T. F. Irvine and P. E. Liley, *Steam and Gas Tables with Computer Equations:* Academic Press Inc., 1984.

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