A THESIS ON

SOME FLOW PROBLEMS IN FLUID DYNAMICS AND MAGNETO-FLUID DYNAMICS

By

V.K. Sud
Department of Mathematics
Indian Institute of Technology
New Delhi

Submitted to the Indian Institute of
Technology, New Delhi for the award
of the Degree of Doctor of Philosophy
in Mathematics
1968

CERTIFICATE

This is to certify that the thesis entitled 'Some Flow Problems in Fluid Dynamics and Magneto-Fluid Dynamics' which is being submitted by Mr. Virender Kumar Sud to the Indian Institute of Technology, Delhi for the award of the Degree of Philosophy in Mathematics, is a record of bonafide research work carried out by him. He has worked under my guidance and supervision for the last three years. His thesis has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full, to any other university or institute for the award of any degree or diploma.

MPSingh 9.2.68

(M.P. Singh)
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-29.

A C K N O W L E D G E M E N T S

I am extremely grateful to Dr. M.P. Singh, M.Sc., Ph.D., Department of Mathematics, Indian Institute of Technology, Delhi for his valuable and constant guidance throughout this work. I express my deep sense of gratitude to Prof. M.K. Jain, M.A., D.Phil., D.Sc., Head of the Department of Mathematics, Indian Institute of Technology, Delhi for his constant encouragement and inspiration. I express my heartfelt and deep gratitude to Dr. G.L. Saini, M.A., Ph.D., Department of Mathematics, Indian Institute of Technology, Delhi for his interest in my progress.

My thanks are also due to the authorities of the Indian Institute of Technology, Delhi for having placed at my disposal all the facilities received during the course of this work.

I will be failing in my duty if I donot thank Mr. G.S. Sekhon, Mr. R.P. Singh, Mr. C.V. Radhakrishna, Mr. J.L. Gupta and Mr. H.R. Sharma for helping me in many ways in the preparation of the thesis. I also offer my sincere thanks to Mr. D.R. Joshi for his commendable work in typing this manuscript.

V.K. Sud

(V.K. Sud)
Department of Mathematics
Indian Institute of Technol
Hauz Khas, New Delhi-29.

S Y N O P S I S

During the last decade and a half, a new science as well as technology has made itself felt, namely Magnetohydrodynamics. It is the study of the motion of an electrically conducting fluid in th presence of a magnetic field. It is fouhd that if a magnetised mediu moves, an electric field is produced. If we consider the medium to be an electrically conducting, the electric field will produce curre These currents interact with magnetic field and produce forces which changes appreciabely the state of motion of the medium. We cannot then use an ordinary Hydrodynamics nor simply ordinary Electrodynamic theory but must study the combination of two which has been given the name Magnetohydrodynamics (with Hydromagnetics, Magnetofluiddynamics as the most common synonyms). Originally, this subject was developed by astrophysicists to study stellar phenomena; the subject is now claimed more by engineers due to its many potential technological applications. Among those we mention the following: Controlled thermonuclear fusion, thrust production for propulsive devices, pumping of liquid metals, high temperature resistant coating, cutting of high temperature alloys and magnetohydrodynamic generatores etc. In recent years, the studies relating to the harmonic oscillations of th mainstream through a body has been of much interest to the expanding field of knowledge which is applicable to turbomachinery, such as fans, compressors, pumps and turbines. The moderate and large scale motions of the atmosphere are greatly influenced by the vorticity of the earth's rotation. In the case of an infinite liquid, rotating as a rigid body about an axis, the amount of energy possessed by the liquid is infinite and is of great interest to know how small disturbances propagate in such a liquid. To understand some of these phenomena, it will be worthwhile to study the flow of a rotating flu around elementary bodies.

The work contained in this thesis has been divided into three main chapters. Chapter I consists of the general survey of the literature in Magnetohydrodynamics. A short survey of the problems discussed also follows. The basic equations of Magnetohydrodynamics for the flow and the Electromagnetic quantities as employed in different situations in the following chapters have been discussed.

The chapter II has been divided into three sections containing problems on fluid flow through an infinite wavy plate. The third and the last chapter contains two sections on flow problems past an oscillating sphere in a compressible viscous fluid.

In section A of chapter II, we consider the fluctuating flow of an electrically conducting fluid past a wavy plate. We take the conductivity of the fluid small so that we can neglect the induced magnetic field. A constant magnetic field is applied at different orientations i.e. in the first part of this section it is parallel to the main stream, while in the second part of this section it is perpendicular to the main stream. The Magnetohydrodynamics equatio are linearized with the assumption of small perturbation in the flov fields. These linearized equations have been solved by introducing the stream function. We have obtained the expressions for the flow fields, the total pressure and the drag. The non-magnetic case has been deduced as a particular case. Several interesting results hav been obtained. The extrema of the disturbance flow fields in the case of uniform free stream are unaffected by the magnetic field and the frequency of the free stream fluctuating flow does not affect the amplitude of the disturbance flow fields in the non-magnetic case, while in the magnetic case the amplitude varies inversely with the frequency. It is found that the magnetic field tends to increase the maximum pressure at the wall. The effect of magnetic field is to produce sinusoidal disturbance in the transverse direction with decreasing amplitude.

In section B of this chapter, we investigate the unsteady motion analogous to the classical Rayleigh problem in the theory of viscous flow. In the magnetic case which we are going to consider, the magnetic viscosity $\frac{1}{\mu_0 \sigma}$ plays the role of kinematic viscosity where μ_0 is the permeability and σ is the conductivity of the fluid. We consider a non-conducting wavy plate of sinusoidal shape at both sides in a conducting, inviscid and incompressible fluid moving with uniform velocity and is suddenly brought to halt in the presence of uniform magnetic field perpendicular to the undisturbed flow. It has been shown that the appropriate initial condition for the flow is the non-magnetic potential flow; the problem has been formulated in terms of stream function for the flow and vector potential for the magnetic field. We have discussed both the cases of configuration i.e. when the phase difference between the upper and lower side is 0(anti-symmetric case) and π(symmetric case). We treat the anti-symmetrical case without thickness while for symmetric configuration of the plate a finite thickness 2δ is taken into account. The unsteady governing equations are linearized using small-perturbation technique and are solved by means of Laplace Transformation. The solutions valid for small and large time has been obtained. The expressions for the pressure and drag at the plate have also been obtained.

In the first part of the section C of this chapter, we study the response of variation of free stream velocity on wall pressure and flow past a wavy wall. If a flat plate is introduced in a shear flow, its effect on the main flow will depend upon several factors, for instance, viscosity of the fluid and the orientation of the plat with respect to the oncoming stream. If the plate is placed paralle to the free stream and if the fluid is inviscid, the presence of the plate will not disturb the main flow, since the slippage of the flui at the surface of the plate is permissible. If however we slightly deform the plate, say for example we consider sinusoidal shape of th plate with small amplitude as compared to its wave length, it will cause an initial discontinuity of the normal component of the veloci at the interface which will be instantaneously propagated into the inviscid fluid, thereby distorting the main flow. Since only small deformation of the flat plate is involved, the corresponding disturbances in the flow fields will be small and hence linearization of t governing equations will be possible. Here we consider several type of free stream flow past the wavy wall: $y = \varepsilon e^{i\lambda x}$ and obtain the resulting flow fields by regular perturbation. Several interesting results have been obtained. It may be noted that the introduction of small constant vorticity does not affect the primary flow due to uniform free stream; however it tends to increase the maximum pressu at the wall. The perturbed flow fields due to the presence of the wave shaped boundary die out transversally at the same rate as in the case of uniform free stream. When the wall is in a parabolic she we notice that the disturbance flow fields decay exponentially in th transverse direction but the rate of decay of velocity fields is slightly faster while that of pressure is slightly slower as compared to the rates of decay of the corresponding fields in the uniform free stream. The extrema of the pressure at the wall coincide with the extrema of the wall. The pressure at the wall or at any point in the fluid incresses as the vorticity parameter a increases.

In part II of the same section, we discuss the problem of wavy wall in a flow with changing direction and magnitude. We consider a special type of free stream flow defined by

$$U_\infty = \frac{U'(y)}{\lambda} e^{i\lambda x}$$

$$V_\infty = - iU(y)e^{i\lambda x}$$

with the restriction that $U(0) = 0$.

It is found that the pressure p_∞ at any point in the fluid due to the main flow is independent of the transverse distance from the wall and its dependence on the longitudinal coordinate is wave-like whose wave length is half that of the wall. The disturbance fields decay exponentially in the transverse direction, the rate of decay being of the order of the wave length of the wall.

The chapter III contains two sections on flow problems on the sphere oscillating in a compressible viscous medium. In section A of this chapter we discuss the slow linear oscillations of a conducting sphere in an electrically conducting, compressible, viscous rotating fluid. Zeroth and first order solutions for the velocity and magnetic fields are obtained in terms of small magnetic Reynolds number. The constant magnetic field H^0 is applied perpendicular to the axis of the rotation of the fluid. The assumption of small oscillations of the sphere justifies to neglect the convective terms in the momentum equation. The solutions valid for the incompressible case have been deduced as a special case and the expression for the hydro- magnetic drag has been obtained. It is found that the magnetic fiel(tends to increase the resistance and decrease the apparent mass of the fluid. The conductivity and viscosity of the fluid tend to increase the drag. It is intersting to note that the rotation offer: no contribution to the drag up to this order. It is found that the contribution to the drag due to Maxwell stress is the frictional for(proportional to the velocity of the sphere while in the case of alig: field problem, there is no contribution of Maxwell stress to the dra; as there are no currents on the surface of the sphere.

In section B of this chapter, we study the linear oscillations of sphere in an infinite mass of compressible fluid. The sphere experiences the variable normal suction or injection and the fluid i: allowed to slip at the surface of the sphere. Here, again, we assum(the slow oscillations of the sphere which helps us in neglecting the convective terms present in the momentum equation. The expressions for the velocity and pressure fields have been obtained. The incompressible case has been deduced from the compressible one by taking the limits $c \to \infty$ and $c^2 s \to p$ where c is the velocity of sound s is the condensation and p is the pressure. The corresponding resu. with no slippage at the boundary have been deduced by taking the limit $\gamma \to 0$, where γ is the slip coefficient. The expressions of drag for various cases have also been obtained. It is found that th slippage tends to decrease the maximum drag while the viscosity and suction tend to increase the maximum drag which depends linearly on the suction parameters.

The numerical computation of the results presented in this thesis has been carried out on the electronic digital computer, ICT 1909 installed at the Indian Institute of Technology, New Delhi.

C O N T E N T S