Energy Convers. Mgmt Vol. 34, No. 12, pp. 1261–1266, 1993

ANALYTICAL THERMAL MODELLING OF MULTI-BASIN SOLAR STILL

G. N. TIWARI,[1] S. K. SINGH[2] and V. P. BHATNAGAR[3]

[1]Centre for Energy Studies, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi-100 016, [2]S.P.M. College, Biharsharif-803 101 and [3]Delhi College of Engineering, Kashmeri Gate, Delhi 6, India

(Received 27 June 1992; received for publication 23 December 1992)

Abstract—Based on energy balances for different components of a multi-basin solar still, namely, the basin-liner, the water mass and the glass cover, analytical expressions for water and glass temperatures have been derived in terms of climatic and design parameters of the system. It is inferred that the daily yield is a maximum for the least water depth in each basin

Solar distillation   Solar energy   Purification of water

NOMENCLATURE

C_w = Specific heat of water (J/kg °C)
C = Constants defined in text
h_{1j} = Total heat transfer coefficients from water mass to glass cover for jth effect (W/m² °C)
h_2 = Total convective and radiative heat transfer coefficient from top glass cover to ambient air (W/m² °C)
h_{3j} = Convective heat transfer coefficient either from basin liner or glass cover to water mass in jth effect (W/m² °C)
h_b = Bottom heat transfer coefficient (W/m² °C)
h_{cwj} = Convective heat transfer coefficient from water to glass cover in jth effect (W/m² °C)
h_{ewj} = Evaporative heat transfer coefficient from water to glass cover in jth effect (W/m² °C)
$I(t)$ = Solar intensity incident on glass cover (W/m²)
K_i = Thermal conductivity of insulating material (W/m² °C)
L = Latent heat of vaporization (J/kg)
L_i = Thickness of insulation (m)
$\dot{m}_{ej}$ = Rate of distillation in jth effect (kg/s)
M_{wj} = Mass of water in jth basin (kg)
P_{gj} = Partial saturated vapour pressure at T_g (N/m²)
P_{wj} = Partial saturated vapour pressure at T_w (N/m²)
Q_{ewj} = Rate of heat flux per m² due to evaporation (W/m²)
t = Time (s)
T_a = Ambient temperature (°C)
T_b = Basin liner temperature (°C)
T_{gj} = Glass cover temperature of jth effect (°C)
T_{wj} = Water temperature of jth effect (°C)
V = wind velocity (m/s)

Greek letters

$(\alpha\tau)_b$ = Fraction of energy absorbed by basin liner
$(\alpha\tau)_w$ = Fraction of energy absorbed by water
ϵ = Emissivity
σ = Stefan–Boltzmann constant (W/m² K⁴)
α, β = Constants defined in text
η = Overall percentage efficiency of still

INTRODUCTION

The working of solar distillation units has been broadly classified into passive and active modes of operation. In a recent review, it is reported that the overall thermal efficiency of a passive distiller is higher than that of an active one due to the lower range of operating temperature [1]. A more detailed classification is given in Table 1. In order to increase the daily output of a conventional distiller, the following effects have been studied:

(i) Dye or charcoal chips [2],
(ii) Back wall as reflector [3, 4],

Table 1. Classification of passive solar still

- Passive solar still
  - Conventional solar still
    - Single effect distiller
      - With dye
      - Without-dye
    - Multi-effect distiller
      - With stationary water
      - With moving water (regenerative still)
        - With feeding
        - Without feeding
  - Inclined solar still
    - Inclined basin solar still
    - Multi-wick still
  - New design solar still
    - Liferaft
    - Spherical
    - Tubular

(iii) Back wall with cotton cloth [5],
(iv) Regenerative effect in back wall and the glass cover [5],
(v) Solar still with internal condenser, Ahmed [6],

and the reported increase in the daily yield of the distiller is about 10–15%. In view of its increased cost of installation and maintenance, it is not appreciable.

The use of latent heat of vaporization for further distillation, which is generally known as double-basin distillation [7, 8], has been shown to increase the daily yield appreciably ($\simeq$35–40%) under a clear climatic condition. Recently, Mahdi [9] has studied the performance of a multibasin solar still and concluded that the latent heat of varporization can be used appreciably only up to the third stage. Mahdi has solved coupled differential equations based upon energy balance equations for different components of the system.

In this communication, an analytical study of a multi-basin solar still (three-stage) has been done in terms of daily yield. Analytical expressions for the temperatures of different water masses and glass covers have been derived using the energy balance equations for the different components of the system. It is inferred that the daily yield is a maximum for the least water depth in each effect.

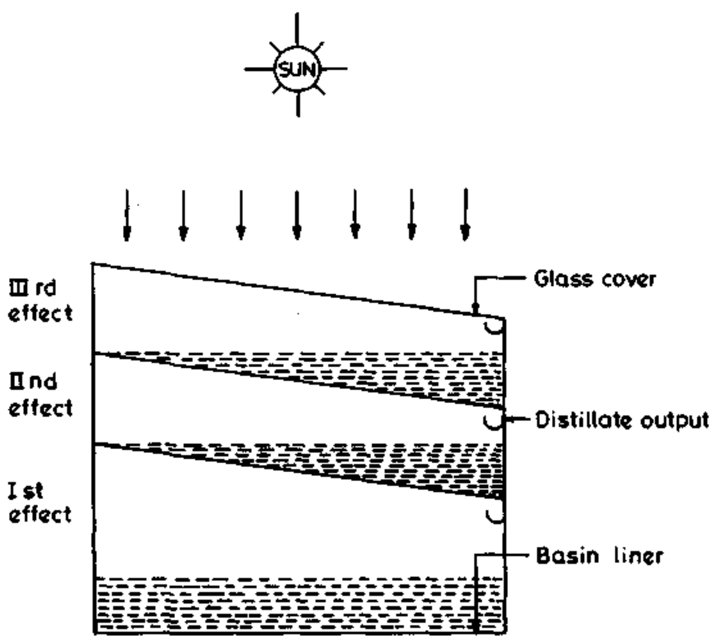

Fig. 1. Cross-sectional view of three-stage distillation unit.

ANALYSIS

A three-stage solar still is shown schematically in Fig. 1. The energy balance equations for the various components are as follows:
For the basin liner

$$(\alpha\tau)_b I(t) = h_{3j}(T_b - T_{wj}) + h_b(T_b - T_a). \quad (1)$$

For the water mass

$$(\alpha\tau)_j I(t) + h_{3j}(T_b - T_{wj}) = M_{wj} C_w (dT_{wj}/dt) + h_{1j}(T_{wj} - T_{gj}). \quad (2)$$

For the glass cover

$$h_{1j}(T_{wj} - T_{gj}) = h_{3(j+1)}[T_{gj} - T_{w(j+1)}] \quad (3)$$

if the glass cover is in contact with water
and

$$h_{1j}(T_{wj} - T_{gj}) = h_2(T_{gj} - T_a) \quad (4)$$

if the glass cover is exposed to ambient.
Here, the subscripts $j = 1, 2, 3$ represent the first, second and third stages, respectively.
Here

$$h_{1j} = h_{rwj} + h_{cwj} + h_{ewj} \quad (5)$$

where

$$h_{rwj} = \{\sigma\epsilon[(T_{wj} + 273)^4 - (T_{gj} + 273)^4]/(T_{wj} - T_{gj})\} \quad (6)$$

$$h_{cwj} = 0.884\{(T_{wj} - T_{gj}) + (P_{wj} - P_{gj})(T_{wj} + 273)/(268.9 \times 10^3 - P_{wj})\}^{1/3} \quad (7)$$

$$h_{ewj} = 0.016 \times h_{cwj} \times (P_{wj} - P_{gj})/(T_{wj} - T_{gj}), \quad (8)$$

$$h_b = [(L_i/K_i) + (1/h_i)]^{-1} \quad (9)$$

and

$$h_2 = 5.7 + 3.8V \text{ [10]}. \quad (10)$$

In order to write the above energy balances, the following assumptions have been made:

(i) The glass covers and the insulating materials have very small heat capacities,
(ii) The temperature remains constant throughout the thickness of the water masses and the glass covers,
(iii) The whole distiller unit is vapour-tight and,
(iv) The glass covers have very small inclination (10–15°) so that their areas are nearly equal to that of the basin liner.

From equation (1),

$$T_b = [(\alpha\tau)_b I(t) + h_{3j} T_{wj} + h_b T_a]/(h_{3j} + h_b). \quad (11)$$

From equation (3)

$$T_{gj} = [h_{1j} T_{wj} + h_{3(j+1)} T_{w(j+1)}]/[h_{1j} + h_{3(j+1)}] \quad (12)$$

when the glass cover is in contact with water
and

$$T_{gj} = (h_{1j} T_{wj} + h_2 T_a)/(h_{1j} + h_2) \quad (13)$$

when the glass cover is exposed to the ambient.

From equations (2), (7), (8) and (9), for $j = 1, 2, 3$, one gets:

$$\left[\frac{dT_{w1}}{dt}\right] + a_1 T_{w1} + a_2 T_{w2} + a_3 T_{w3} = f_1(t) \quad (14)$$

$$\left[\frac{dT_{w2}}{dt}\right] + a_1' T_{w1} + a_2' T_{w2} + a_3' T_{w3} = f_2(t) \quad (15)$$

$$\left[\frac{dT_{w3}}{dt}\right] + a_1'' T_{w1} + a_2'' T_{w2} + a_3'' T_{w3} = f_3(t) \quad (16)$$

where

$$a_1 = (U_{12} + U_{1b})/(M_{w1} C_w)$$

$$a_2 = -U_{12}/M_{w1} C_w, \quad a_3 = 0$$

$$f_1(t) = \left[(\alpha\tau)_1 I(t) + \frac{h_{31}(\alpha\tau)_b}{(h_{31} + h_b)} I(t) + U_{1b} T_a\right] \Big/ M_{w1} C_w$$

$$a_1' = -U_{12}/M_{w2} C_w, \quad a_2' = (U_{12} + U_{13})/M_{w2} C_w$$

$$a_3' = -U_{13}/M_{w2} C_w, \quad f_2(t) = (\alpha\tau)_{II} I(t)/M_{w2} C_w$$

$$a_1'' = 0, \quad a_2'' = -U_{13}/M_{w3} C_w$$

$$a_3'' = (U_{13} + U_{32})/M_{w3} C_w$$

$$f_3(t) = [(\alpha\tau)_{III} I(t) + U_{32} T_a]/M_{w3} C_w$$

$$U_{12} = h_{11} h_{32}/(h_{11} + h_{32})$$

$$U_{1b} = h_{31} h_b/(h_{31} + h_b)$$

and

$$U_{32} = h_{13} h_2/(h_{13} + h_2). \quad (17)$$

Multiplying equation (15) by α and equation (16) by β and adding to equation (14)

$$[d(T_{w1} + \alpha T_{w2} + \beta T_{w3})/dt] + (a_1 + \alpha a_1' + \beta a_1'')T_{w1} + (a_2 + \alpha a_2' + \beta a_2'')T_{w2} + (a_3 + \alpha a_3' + \beta a_3'')T_{w3} = f_1(t) + \alpha f_2(t) + \beta f_3(t)$$

or

$$[d(T_{w1} + \alpha T_{w2} + \beta T_{w3})/dt] + C(T_{w1} + \alpha T_{w2} + \beta T_{w3}) = f_1(t) + \alpha f_2(t) + \beta f_3(t) \quad (18a)$$

where

$$C = a_1 + \alpha a_1' + \beta a_1'' \quad (18b)$$

$$C\alpha = a_2 + \alpha a_2' + \beta a_2'' \quad (18c)$$

$$C\beta = a_3 + \alpha a_3' + \beta a_3''. \quad (18d)$$

From equations (18c) and (18d), one gets

$$\alpha = [a_2'' a_3 - a_2(a_3'' - C)]/\{[(a_2' - C)(a_3'' - C)] - a_2'' a_3'\} \quad (18e)$$

$$\beta = [a_2 a_3' - a_3(a_2' - C)]/[(a_2' - C)(a_3'' - C) - a_2'' a_3']. \quad (18f)$$

Using equations (18e) and (18f) in equation (18b)

$$C^3 - A_1 C^2 + A_2 C + A_3 = 0 \quad (19a)$$

where

$$A_1 = a_1 + a_2' + a_3'' \quad (19b)$$

$$A_2 = [(a_1 a_2' - a_2 a_1') + (a_2' a_3'' - a_3' a_2'') + (a_3'' a_1 - a_1'' a_3)] \quad (19c)$$

$$A_3 = [a_1(a_2'' a_3' - a_2' a_3'') + a_1'(a_2 a_3'' - a_3 a_2'') + a_1''(a_2' a_3 - a_3' a_2)]. \quad (19d)$$

Solutions to equation (19a) are

$$C_1 = [(-D_1/4) + \sqrt{(D_1^2/4) + (E_1/27)}]^{1/3} + [-D_1/4) - \sqrt{(D_1^2/4) + (E_1/27)}]^{1/3} + (A_1/3) \quad (20a)$$

$$C_2, C_3 = \{-(C_1 - A_1)/2 \pm \sqrt{(C_1 - A_1)^2/4 - [A_2 + (C_1 - A_1)C_1]}\} \quad (20b)$$

where

$$D_1 = (-2A_1^3/27) + (A_1 A_2/3) + A_3 \quad (20c)$$

$$E_1 = A_2 - (A_1^2/3). \quad (20d)$$

Multiplying equation (18a) e^{Ct} and integrating between 0 and t, one gets its solution as:

$$T_{w1} + \alpha T_{w2} + \beta T_{w3} = \frac{\overline{f_1(t)} + \alpha\overline{f_2(t)} + \beta\overline{f_3(t)}}{C}(1 - e^{-Ct}) + (T_{w_{10}} + \alpha T_{w_{20}} + \beta T_{w_{30}})e^{-Ct} \quad (21)$$

where $\overline{f_1(t)}$ etc. and $T_{w_{10}}$ etc., are the averaged values between the interval 0 and t and the values at $t = 0$, respectively.

The average water temperature can be obtained as

$$\overline{T_{w1}} + \alpha_j\overline{T_{w2}} + \beta_j\overline{T_{w3}} = \frac{1}{t}\int_0^t (T_{w1} + \alpha_j T_{w2} + \beta_j T_{w3})\,dt = \left[\frac{\overline{f_1(t)} + \alpha_j\overline{f_2(t)} + \beta_j\overline{f_3(t)}}{C_j}\right]$$
$$\times\left(1 - \frac{1 - e^{-C_j t}}{C_j t}\right) + (T_{w_{10}} + \alpha_j T_{w_{20}} + \beta_j T_{w_{30}})\left(\frac{1 - e^{-C_j t}}{C_j t}\right) = \overline{F_j(t)} \text{ (supposed)}. \quad (22)$$

Here, $j = 1, 2, 3$ correspond to the three values of C, α and β as given in equations (20a), (20b), (18e) and (18f).

Solutions to equations (22), for $j = 1, 2, 3$, are

$$\overline{T_{w1}} = \overline{F_1(t)} - \alpha_1\overline{T_{w2}} - \beta_1\overline{T_{w3}} \quad (23a)$$

$$\overline{T_{w2}} = \frac{(\beta_1 - \beta_2)[\overline{F_2(t)} - \overline{F_3(t)}] - (\beta_2 - \beta_3)[\overline{F_1(t)} - \overline{F_2(t)}]}{(\beta_1 - \beta_2)(\alpha_2 - \alpha_3) - \beta_2\beta_3(\alpha_1 - \alpha_2)} \quad (23b)$$

$$\overline{T_{w3}} = \frac{(\alpha_2 - \alpha_3)[\overline{F_1(t)} - \overline{F_2(t)}] - (\alpha_1 - \alpha_2)[\overline{F_2(t)} - \overline{F_3(t)}]}{(\alpha_2 - \alpha_3)(\beta_1 - \beta_2) - (\alpha_1 - \alpha_2)(\beta_2 - \beta_3)}. \quad (23c)$$

Using equations (23a), (23b) and (23c) the average temperature of the glass covers can be obtained from equations (12) and (13).

Knowing the temperatures of the glass covers and the water masses, the rate of heat flux per m^2 due to evaporation can be obtained from the following equation

$$Q_{ewj} = h_{ewj}(T_{wj} - T_{gj}) \quad (24)$$

where h_{ewj} is given by equation (8).

The rate of distillation per m^2 is

$$\dot{m}_{ej} = \frac{Q_{ewj}}{L}. \quad (25)$$

The overall thermal efficiency is

$$\eta = \frac{\sum Q_{ewj}}{\sum I(t)\,\Delta t} \times 100. \quad (26)$$

RESULTS AND DISCUSSION

For the efficiency as well as the daily yield of the still to be high, according to equations (26) and (24), the water temperatures T_{wj} should be high. This requires that the l.h.s. of equation (22), which gives the linear sum of the average water temperatures, should be high. Considering the following cases:

Case (i) $C_j t \ll 1$:

Under this condition $[(1 - e^{-C_j t})/C_j t] \to 1$, and the r.h.s. of equation (22) becomes independent of the system and the climatic parameters, and hence, there is no yield.

Case (ii) $C_j t \gg 1$:

Under this condition $[(1 - e^{-C_j t})/C_j t] \rightarrow 0$, and the r.h.s. of equation (22) is a maximum and strongly dependent upon the system and the climatic parameters. Thus, for the daily yield to be high, C_j should be large, for a given t (1 h). This requires, according to equation (18b) that the values of the a's should be high, and hence, the water mass M_w, i.e. the water depth in each basin should be small. If M_{w3}, the water mass in the third effect, is large, it is obvious from equations (16) and (17), that T_{w3} becomes constant and nearly equal to the ambient temperature. This is the case of $\beta = 0$, when the third stage gives no yield and the unit becomes just a two-effect one. And if M_{w2} is also very large, the second effect also becomes inoperative as T_{w2}, equal to the ambient temperature, and the unit becomes equivalent to a single-basin system.

Acknowledgement—The authors are grateful to Professor P. B. Sharma, Principal, Delhi College of Engineering, Delhi for his continuous support during the research work reported in the paper.

REFERENCES

1. G. N. Tiwari, Recent advances in solar distillation. In *Solar Energy and Energy Conservation* (Edited by Raj Kamal *et al.*), Chap. II, pp. 32–149. Wiley Eastern, New Delhi (1992).
2. M. S. Sodha, A. Kumar, G. N. Tiwari and G. C. Pandey, *Appl. Energy* **7**, 147 (1980).
3. P. Bivulswas and S. Tadtiam, *Int. Symposium Workshop on Renewable Energy Sources—Lahore*. Elsevier, Amsterdam (1984).
4. A. Tamini, *Int. J. Sol. Wind Technol.* **4**, 443 (1987).
5. P. Wibulswas, *Renewable Energy Rev. J.* **6**, 101 (1984).
6. S. T. Ahmed, *Sol. Wind Technol.* **5**, 637 (1988).
7. M. S. Sodha, J. K. Nayak, G. N. Tiwari and A. Kumar, *Energy Convers. Mgmt* **20**, 23 (1980).
8. O. C. Iloeje, *Instersol 85*, Vol. 2, p. 1339. Pergamon Press, Oxford (1985).
9. N. A. Mahdi, *Energy* **17**, 87 (1992).
10. J. A. Duffie and W. A. Beckman, *Solar Engineering of Thermal Processes*. Wiley, New York (1980).