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PERFORMANCE EQUATIONS OF A COLLECTOR CUM STORAGE SYSTEM USING PHASE CHANGE MATERIALS

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Abstract—A thermal analysis has been developed for a collector cum system for quasi-steady-state conditions using phase change materials. Performance equations of the Hottel–Whillier–Bliss type for flat-plate collector cum storage system have been obtained. Calculations have been performed for a wide range of parameters to investigate the applicability of the developed mathematical model.

1. INTRODUCTION

Most solar water-heating systems employ water for sensible thermal energy storage in a separate storage tank for use during off sunshine hours. Commercialization of batch solar water heaters combining collection and storage in the same unit is still restrictive and unpopular because of the bulky size and various other problems. The reduction of the storage size is at least conceptually feasible by using phase change materials (PCM). Buddhi *et al.*[1] have designed and tested such a solar collector cum storage system under varying conditions of design parameters using stearic acid as the PCM.

A flat-plate collector without storage has been very well studied and its performance equations—known as Hottel–Whillier–Bliss equations (HWB)[2,3]—are well characterized. If a PCM is, however, used in a flat-plate collector as a storage medium, the heat transfer becomes complicated and there is no possibility to use the HWB equations in their present form for characterization of the system.

Employment of a PCM in a flat-plate collector further dampens the rate at which various temperatures in the system change and hence one can still assume the system to be in a quasi-steady-state condition. The corresponding heat transfer equations of a PCM collector cum storage system therefore now take the form of the Stefan problem of moving boundary heat transfer. In this communication paper a detailed mathematical model has been developed for such a system and modified forms of the HWB equations have been obtained. Different sets of equations have to be used for the charging and discharging mode because the melting during charging and the freezing during discharging occurs at the interface between the absorbing plate and the PCM. Performance for a variety of parameters has been obtained for typical New Delhi climatic conditions.

2. ANALYSIS

The energy balance equations for the system have to be written separately for the charging and discharging mode by making the following assumptions.

1. The system can be assumed to be in the quasi-steady-state since the variations in temperatures are slow.
2. Convection is the dominant mode of heat transfer between the absorbing plate and the liquid PCM during the charging mode.
3. Liquid PCM is just below the absorbing plate during the charging mode.
4. The temperature of the melted zone remains constant (equal to the melting temperature).
5. Construction is of sheet and parallel tube type.
6. The headers provide uniform flow to tubes.
7. The covers are opaque to infrared radiation.
8. Thermophysical properties of all the materials are independent of the temperature.
9. Shading of the collector absorber plate is negligible.
10. During the discharging mode, the melted zone is between two solidified zones of the PCM.

2.1 *Charging mode*

2.1.1 *Temperature distribution and modified form of HWB equation*. The schematic of the collector cum storage system is given in Fig. 1(a). As soon as the solar radiation is incident on the absorber plate its temperature starts rising and when it reaches the melting point of the PCM, the melting zone is just below the absorbing plate. The temperature of the absorbing plate is maximum in between the fluid channels and decreases steadily toward the fluid channel. Considering symmetry, taking the origin in between the fluid channels and assuming an element of length x and unit width (parallel to the length and fluid flow) somewhere between the origin and the fluid channel, one can write down the following energy balance equation

$$S\Delta x = k\delta \frac{dT}{dx} - k\delta \frac{d}{dx}\left(T + \frac{\partial T}{\partial x}\Delta x\right) + U_{u}(T - T_{a})\Delta x + h_{c}(T - T_{m})\Delta x. \quad (1)$$

(Rate of absorbed energy by the absorber; rate of heat conducted into the element; rate of heat conducted away from the element; rate of heat loss from the plate to the ambient; rate of heat transferred from absorber to PCM) where S is the absorbed solar radiation, k is

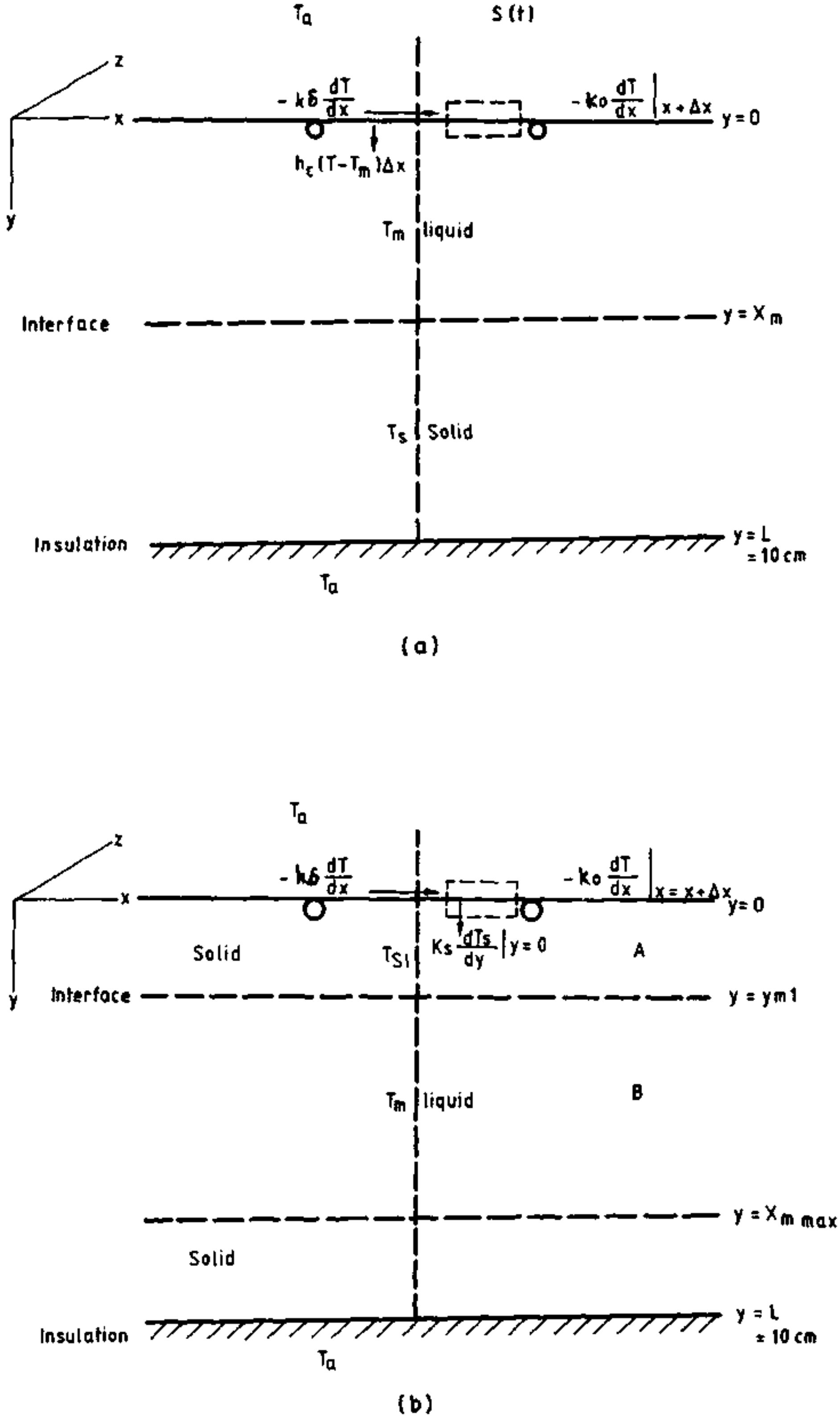

Fig. 1. Energy balance on fin element during the charging and discharging mode.

the thermal conductivity of the absorber, δ is the thickness of the absorber, T_a is the ambient temperature (°C), U_{tt} is the top heat loss coefficient (from the absorber plate to the ambient), h_c is the heat transfer coefficient between the absorber and the PCM, T_m is the melting temperature of the PCM, and $T(x)$ is the temperature distribution between the two tubes.

Following Duffie and Beckmann[4], eqn (1) can be written in the form of the following differential equation,

$$\frac{d^2T}{dx^2} = \frac{U_{t1} + h_c}{k\delta}\left[T - T_a - \frac{S + h_c(T_m - T_a)}{U_{t1} + h_c}\right]. \quad (2)$$

The difference between the usual flat-plate collector equation and the equation for PCM collector cum storage system lies in the replacement of overall heat loss U_L by $(U_{t1} + h_c)$ which is the combined effect of heat loss from the top and heat gain to the PCM for storage. The latter manifests itself in an increase of X_m, the melting zone that assumes its time variation according to the equation developed in the next section.

2.1.2 *Temperature distribution in the solid PCM: moving boundary problem*. The rate at which the energy $h_c(T - T_m)$ is transferred to the PCM is partially utilized in melting the solid–liquid interface (storage of energy) and it is partially utilized in transferring the energy to the solid PCM. Mathematically

$$h_c(T - T_m) = H\rho_l \frac{dX_m(t)}{dt} + h_s(T_m - T_s(y, t)|_{y=X_m}). \quad (3)$$

(Rate of heat transfer from absorber plate to PCM; rate of heat energy stored in PCM; rate of energy transferred from the liquid PCM to the solid PCM). Since the heat transfer is predominantly one dimensional, the temperature distribution in the solid PCM is given by the following Fourier heat conduction equation,

$$\frac{\partial^2 T_s(y, t)}{\partial y^2} = \frac{\rho_s c_p}{k_s}\frac{\partial T_s(y, t)}{\partial t}. \quad (4)$$

The temperature distribution $T_s(y, t)$ in the solid phase change material has to satisfy the following boundary conditions

$$-k_s \left.\frac{\partial T_s}{\partial y}\right|_{y=X_m} = h_s(T_m - T_s|_{y=X_m}) \quad (5)$$

and

$$-k_s \left.\frac{\partial T_s}{\partial y}\right|_{y=L} = h_a(T_s|_{y=L} - T_a). \quad (6)$$

We make a bold assumption that the temperature distribution inside the solid PCM is periodic with the same fundamental frequency as the variation of ambient temperature, i.e.,

$$T_s(y, t) = A_0 + A_1 y + \mathrm{Re} \sum_{n=1}^{\infty} [A_n \exp(\alpha_n y) + A_{n_1} \exp(-\alpha_n y)] \exp(in\omega t) \quad (7)$$

where $\alpha_n = (1 + i)\sqrt{(n\omega\rho_s C_s/2k_s)}$ and A_0, A_1, A_n, and A_{n_1} are unknown constants to be found by the boundary conditions (5) and (6). If ω in eqns (7) and (8) is taken to be the frequency varying over a period of 24 hours, then one has to be careful to match the temperature at the boundaries when we change from the charging mode to the discharging mode.

2.1.3 *Useful collected energy*. The rate of useful energy collection and the corresponding hot water temperature are obtained by writing the energy balance equation over an element of the fluid channel in the direction of fluid flow (Fig. 1), i.e.,

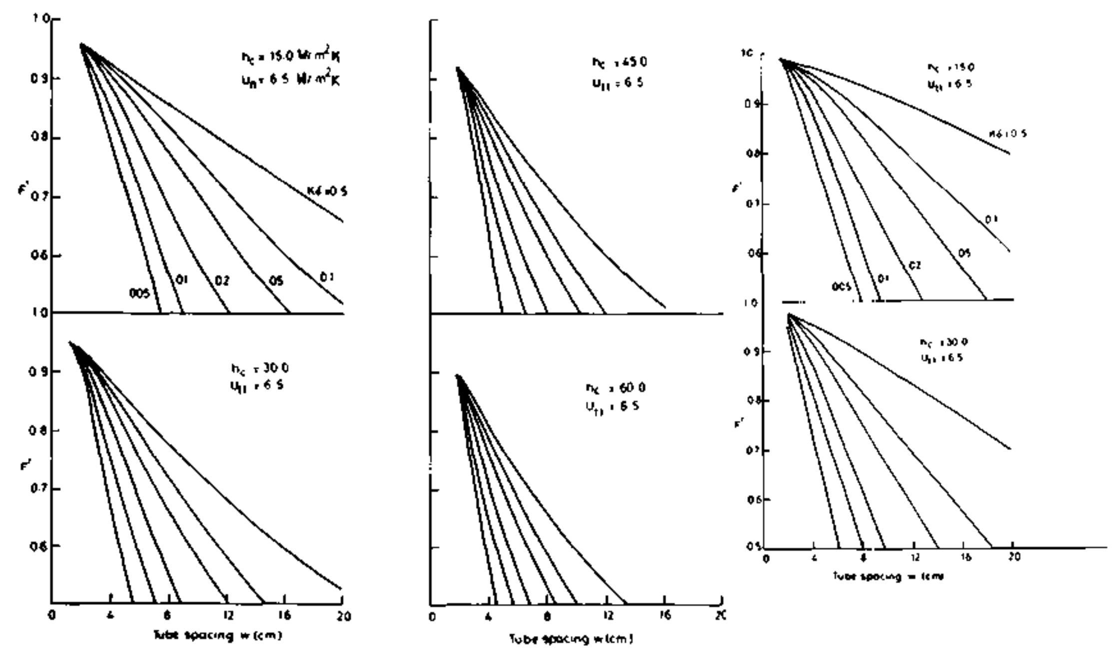

Fig. 2. Collector efficiency factor F' as a function of tube spacing for 12 mm diameter tubes, h_f = 350 W/m^2 K.

$$-\frac{\dot{m}c_p}{n_c}\frac{\partial T_f}{\partial z}\Delta z = q_u \Delta z \tag{8}$$

where q_u has to be obtained from the eqn (13).

2.2 *Solution of equations*

2.2.1 *Heat transfer along the fin.* Following Duffie and Beckman[4] one can define the following parameters in eqn (2).

$$m^2 = \frac{U_{t1} + h_c}{k\delta} \quad \text{and}$$

$$\psi = T - \left\{\frac{S + h_c(T_m - T_a)}{U_{t1} + h_c}\right\}. \tag{9}$$

Using the same analysis one can arrive at the following equation for the energy collected on both sides of the tube, i.e., (after rearranging)

$$q_{fin} = F(w - d)S + [h_c(T_m - T_a) - (U_{t1} + h_0)(T_b - T_a)]. \tag{10}$$

F, the fin efficiency factor, has a lower value in this case than the normal flat-plate collector because $m(w - d)/2$ is about three times more than the normal flat-plate collector for the same tube spacing. However, the fin efficiency can be increased by keeping the tubes closer which ultimately affects the storage capacity.

The useful gain of the collector also includes the energy collected above the tube region that is used only by the absorber plate. The equation is

$$q_{tube} = d[S + h_c(T_m - T_a) - (U_{t1} + h_c)(T_b - T_a)]; \tag{11}$$

and the total useful gain for the collector per unit of length in the flow direction is the sum of eqns (10) and (11), i.e.,

$$q_u = [F(w - d) + d][s + h_c(T_m - T_a) - (U_{t1} + h_c)(T_b - T_a)]. \tag{12}$$

It is easy to arrive at the following equation

$$q_u = wF'[S + h_c(T_m - T_a) - (U_{t1} + h_c)(T_f - T_a)] \tag{13}$$

where

$$F' = \frac{1/(U_{t1} + h_c)}{\left[\frac{1}{(U_{t1} + h_c)[(w - d)F + d]} + \frac{1}{C_b} + \frac{1}{\pi h_{fi} d_i}\right]}. \tag{14}$$

F' is the collector efficiency factor and has lower values than the normal flat-plate collector because $(U_{t1} + h_c)$ is about ten times more than U_L for the normal flat-plate collector (Fig. 2). However, there must be a compromise between collection and storage. The lower value of F' is compensated for by the stored energy calculated by the magnitude of the melting zone, i.e., for X_m. In accordance with eqn (13) the modified form of the HWB equation expressing the instantaneous efficiency of a collector with PCM can be written in the form

$$\eta = F'\left[1 - \frac{(U_{t1} + h_c)(\bar{T}_f - T_a)}{S + h_c(T_m - T_a)}\right]. \tag{15}$$

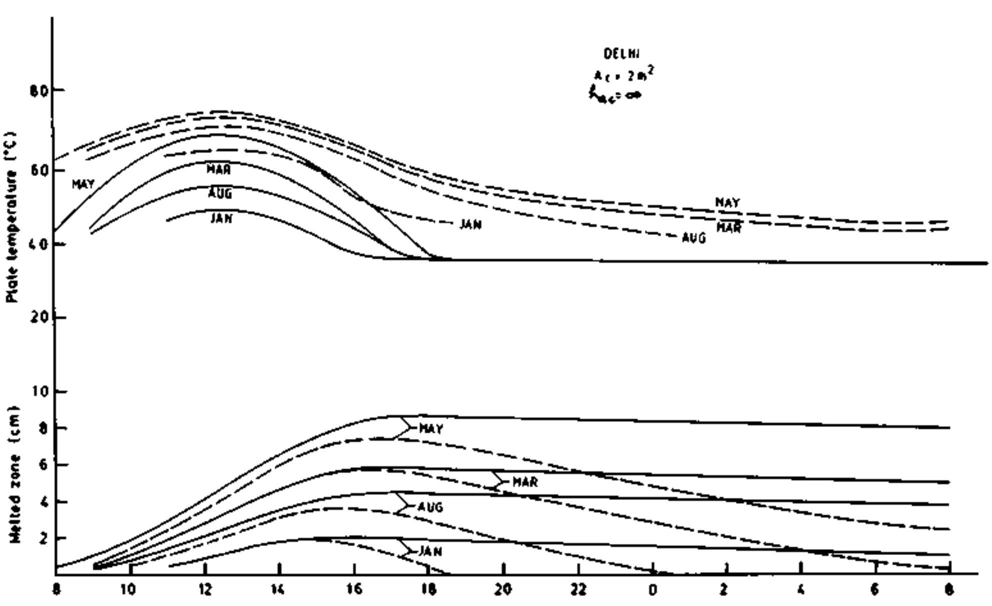

Fig. 3. Hourly variation of melted zone and stagnation (plate) temperature with and without insulating cover.

One can approximate the mean fluid temperature $\bar{T}_f$, by

$$\frac{T_{fo} + T_{fi}}{2}.$$

2.2.2 *Heat transfer in the PCM*. To evaluate the constants A_0, A_1, A_n, and A_{n1} of eqn (7), the boundary conditions are given in eqns (5) and (6) and the expressions for the constants are obtained as

$$A_0 = \frac{(k_s + h_a L)h_s T_m - (h_s X_m - k_s)h_s T_{ao}}{h_s(k_s + h_a L) - h_a(h_s X_m - k_s)}, \quad (16)$$

$$A_1 = \frac{h_a h_s (T_{ao} - T_m)}{h_s(k_s + h_a L) - h_a(h_s X_m - k_s)}, \quad (17)$$

$$A_n = \frac{h_a T_{an}(h_s + k_s \alpha_{n1})\exp(-\alpha_n X_m)}{(h_s - k_s\alpha_n)(k_s\alpha_n - h_a) + (h_s + k_s\alpha_n)(h_a + k_s\alpha_n)}, \quad (18)$$

and

$$A_{n1} = \frac{h_a T_{an}(h_s - k_s\alpha_n)\exp(\alpha_n X_m)}{(h_s - k_s\alpha_n)(k_s\alpha_n - h_a) + (h_s + k_s\alpha_n)(h_a + k_s\alpha_n)\exp(\alpha_n(X_m - L))}. \quad (19)$$

Using the value of T from the solution of eqn (2) and $T_s(y, t)$ from eqn (7) one gets a nonlinear differential equation for X_m [eqn (3)]. Therefore, eqn (3) is solved by using an iterative approach and assuming that $(k_s \delta T_s/\delta y)|_{y=X_m} = h_s(T_m - T_s|_{y=X_m}) = C$ is constant for some small arbitrary time interval. Constants A_0, A_1, A_n, A_{n1} are first calculated by taking $X_m = 0.0$. The resulting expression for T_s [eqn (7)] is substituted in eqn (3) and a value of X_m is found from eqn (3). With this value of X_m, T_s is again calculated at the new X_m. The procedure is repeated until the two consecutive values of X_m match within 1%. The general expression for X_m can be put in the following form

$$X_m(T_n) = X_m(T_{n-1}) + [h_c(T - T_m) + C]t/H\rho_1. \quad (20)$$

2.2.3 *Useful collected energy and fluid temperature*. Using eqn (10) in eqn (7) and solving the resulting differential equation with $T_f = T_{fi}$ (at $z = 0$), one gets

$$\frac{T_{fo} - T_a - \left\{\dfrac{S + h_c(T_m - T_a)}{U_{t1} + h_c}\right\}}{T_{fi} - T_a - \left\{\dfrac{S + h_c(T_m - T_a)}{U_{t1} + h_c}\right\}} = \exp\left(\frac{-(U_{t1} + h_c)F'A_c}{\dot{m}c_p}\right). \quad (21)$$

Again following Duffie and Beckman[4], one can easily get

$$\eta = F_R\left[1 - \left\{\frac{(U_{t1} + h_c)(T_i - T_a)}{S + h_c(T_m - T_a)}\right\}\right] \quad (22)$$

where

$$F_R = \frac{\dot{m}c_p}{A_c(U_{t1} + h_c)}\left[(T_{fo} - T_{fi}) \Big/ \left\{\frac{S + h_c(T_m - T_a)}{U_{t1} + h_c} - (T_{fi} - T_a)\right\}\right]. \quad (23)$$

Equations (15) and (22) are exactly analogous to the familiar HWB equations with the solar radiation term S replaced by the term $S + h_c(T_m - T_a)$, the total energy responsible for heat transfer along the fin.

2.3 *Discharging mode*

When the rate of useful energy gain becomes less than the rate of thermal losses, PCM starts to freeze and the moving boundary problem can be solved according to the geometry shown in Fig. 1(b) in which the melted zone is between two solidified zones of the PCM.

2.3.1 *Temperature distribution and modified form of HWB equations*. Considering an element of length x and of unit width (parallel to the length and fluid flow), one can write the following energy balance equation

$$k_s \frac{\partial T_{s1}}{\partial y}\bigg|_{y=0} \Delta x = U_{t2}(T - T_a)\Delta x + k\delta\frac{\partial T}{\partial x} - k\delta\frac{\partial}{\partial x}\left(T + \frac{\partial T}{\partial x}\Delta x\right). \quad (24)$$

(Rate of heat transferred from PCM to absorber plate; rate of heat loss from the plate to the ambient; rate of heat conducted to the element; rate of heat conducted away from the element).

The resulting differential equation now becomes

$$\frac{d^2T}{dx^2} = \frac{U_{t2} + C_2}{k\delta}\left[T - T_a - \left(\frac{C_1 T_m - C_2 T_a}{U_{t2} + C_2}\right)\right], \quad (25)$$

where

$$C_1 = \frac{h_{ac} h_1 k_s}{h_1 k_s + h_{ac}(h_1 Y_{m1} + k_s)},$$

$$C_2 = \frac{h_{ac}^2(k_s + h_1 Y_{m1})}{h_1 k_s + h_{ac}(h_1 Y_{m1} + k_s)}.$$

The second order differential equation, eqn (25), is solved similarly to eqn (2) and one can arrive at the following equation for the rate of useful heat gain,

$$q_u = WF'[(C_1T_m - C_2T_a) - (U_{t2} + C_2)(T_f - T_a)]. \quad (26)$$

2.3.2 *Temperature distribution in the solid PCM: moving boundary problem*. At the solid–liquid interface for the region A–B, the energy balance equation can be written as

$$H\rho_1 A_c \frac{\partial Y_{m1}}{\partial t} = A_c h_1 (T_m - T_s|_{y=Y_{m1}}) + U_b A_s (T_m - T_a). \quad (27)$$

The temperature distribution in the solidified zone is again assumed to be periodic [eqn (9)] with the constants to be determined by the following boundary conditions

$$k_s \frac{\partial T_{s1}}{\partial y}\bigg|_{y=Y_{m1}} = h_1(T_m - T_s|_{y=Y_{m1}}), \quad (28)$$

$$k_s \frac{\partial T_{s1}}{\partial y}\bigg|_{y=0} = h_{ac}(T_s|_{y=0} - T). \quad (29)$$

Equation (27) is solved in the same way as before, i.e.,

$$Y_{m1}(T_n) = Y_{m1}(T_{n-1}) + [A_c h_1 (T_m - T_{s1}|_{y=Y_{m1}}) + U_b A_s (T_m - T_a)]/H\rho_1. \quad (30)$$

As previously, the useful collected energy is obtained by the usual expression $\dot{m}C_f(T_{fo} - T_{fi})$ with T_{fo} given by

$$\frac{T_{fo} - T_a - \left[\dfrac{C_1T_m - C_2T_a}{U_{t2} + C_2}\right]}{T_{fi} - T_a - \left[\dfrac{C_1T_m - C_2T_a}{U_{t2} + C_2}\right]} = \exp\left(\frac{-F'A_c(U_{t2} + C_2)}{\dot{m}C_p}\right). \quad (30)$$

3. RESULTS AND DISCUSSIONS

A collector cum storage system using phase change material can be analyzed in terms of equations (15) and (22) which are of the same type as HWB equations for a flat-plate collector. The equivalent HWB equations depend on the melting temperature of PCM and heat transfer coefficiency (h_c) between the absorber plate and liquid PCM. In modified HWB equations, the solar radiation term S has been replaced during the charging by the term $[S + h_c(T_m - T_a)]$. During discharging it is replaced by the term $(C_1T_m - C_2T_a)$, where C_1 and C_2 are related to the air conductancy and the heat transfer coefficient between the melting zone and the freezing zone. In PCM collector equation (2), the overall heat loss coefficient (U_L) of a normal flat-plate collector has been replaced by $(U_{t1} + h_c)$ which is the combined effect of heat loss from the top and heat gain to the PCM. Fin efficiency factor F and collector efficiency factor F' of the PCM collector have lower values than a flat-plate collector [eqns (10) and (14)] as shown in Fig. 2 because of the storage effect. The calculations have been performed by taking stearic acid as the PCM material; the thermodynamic properties are given in Table 1. One has to compromise between the energy collection and storage. Generally the volume of a PCM decreases when the freezing occurs and an air cavity is formed between the absorber and the PCM solid layer. For a collector cum storage system using stearic acid as PCM, the performance curves for a number of parameters are plotted. The parametric variation includes the air conductance between the absorber plate and the PCM (during the discharging period), the fluid flow rate, heat loss coefficient, and the heat transfer coefficient. The one distinction in the behavior of these curves for a conventional flat-plate collector[3]: the efficiency factor drops down much faster with the increasing fin distance. In such systems therefore, the tubes have to be kept very close to each other even in the case of high conducting material for the absorber plate.

The system has been studied for a time period of 24 hours. The quantity of interest is therefore the time variation of the plate temperature and the outlet fluid temperature. The following sets of calculations have been performed for the climatic variations of Delhi (Latitude 28°55′N); the ambient temperature and the solar radiation data are given in Table 2.

1. Time variation of the plate temperature with no insulating cover on the glazing during off sunshine hours and with no air gap between the absorbing plate and the PCM, i.e., $h_{ac} \rightarrow \infty$;
2. time variation of the plate temperature with insulating cover [$U_t \approx 0.5(W/m^2\,K)$] over the glazing during off sunshine hours and with no air gap between the absorbing plate and the PCM, i.e., $h_c \rightarrow \infty$;
3. time variation of plate temperature with insulating cover over the glazing during off sunshine hours and with an air gap between the absorbing plate and the PCM, $h_{ac} = 5$ W/m^2 K.

Table 1. Thermophysical properties of stearic acid (PCM)[5,6]

Melting temperature, T_m (°C)	Thermal conductivity, k_s (W/m K)	Specific heat capacity C_p (kJ/kg K)	Latent heat of fusion (kJ/kg)	Density ρ_s (kg/m^3)	e
58.1	0.29	1.590	169.0	847.0	965.0

Table 2. Hourly variation of solar radiation and ambient temperature during the representative months in Delhi (Latitude 28°55')

	Solar radiation (kW/m²)				Ambient temperature (°C)			
Time	January	March	May	August	January	March	May	August
0100	0.0	0.0	0.0	0.0	9.1	17.1	28.4	27.1
0200	0.0	0.0	0.0	0.0	8.4	16.3	27.7	26.7
0300	0.0	0.0	0.0	0.0	7.9	15.7	27.2	26.4
0400	0.0	0.0	0.0	0.0	7.4	15.3	26.7	26.2
0500	0.0	0.0	0.0	0.0	7.3	15.1	26.6	26.1
0600	0.0	0.001	0.03	0.012	7.6	15.4	26.9	26.3
0700	0.0	0.066	0.177	0.101	8.3	16.2	27.6	26.6
0800	0.090	0.26	0.383	0.243	9.5	17.5	28.8	27.3
0900	0.271	0.475	0.581	0.392	11.4	19.5	30.6	28.3
1000	0.433	0.655	0.746	0.512	13.5	21.7	32.7	29.4
1100	0.556	0.781	0.865	0.607	15.8	24.3	35.1	30.7
1200	0.618	0.845	0.925	0.640	18.1	26.7	37.3	32.0
1300	0.615	0.844	0.920	0.659	19.8	28.5	39.0	32.9
1400	0.552	0.769	0.844	0.595	20.9	29.7	40.1	33.5
1500	0.430	0.635	0.719	0.510	21.3	30.2	40.5	33.7
1600	0.268	0.458	0.547	0.392	20.9	29.7	40.1	33.5
1700	0.099	0.249	0.352	0.255	19.9	28.7	39.1	32.9
1800	0.005	0.064	0.158	0.107	18.4	27.0	37.6	32.1
1900	0.0	0.001	0.027	0.013	16.5	25.1	35.8	31.1
2000	0.0	0.0	0.0	0.0	14.7	23.1	34.0	30.1
2100	0.0	0.0	0.0	0.0	13.2	21.4	32.4	29.1
2200	0.0	0.0	0.0	0.0	11.8	19.9	31.0	28.5
2300	0.0	0.0	0.0	0.0	18.7	18.7	29.9	27.9
2400	0.0	0.0	0.0	0.0	9.8	17.8	29.1	27.5

4. The above set of calculations were performed for the outlet fluid temperature from the system. Some of the results are shown in Figs. 3, 4, and 5. These figures also show the variation of the depth of the melting zone with time.

It is seen that the plate temperature without air conductance and with an insulating cover during the discharging mode is nearly equal to the melting temperature of the PCM. For finite conductances, however, the plate temperature drops steeply during the discharging mode. Figure 5 shows that the water temperature is nearly equal to the melting temperature of the PCM if the air conductance between the absorber plate and the solid PCM layer is infinite and the collector area is sufficient and if an insulating cover is used during the discharging period. An air gap between the solidified PCM and the absorber, however, reduces the water outlet temperature by about 10°C.

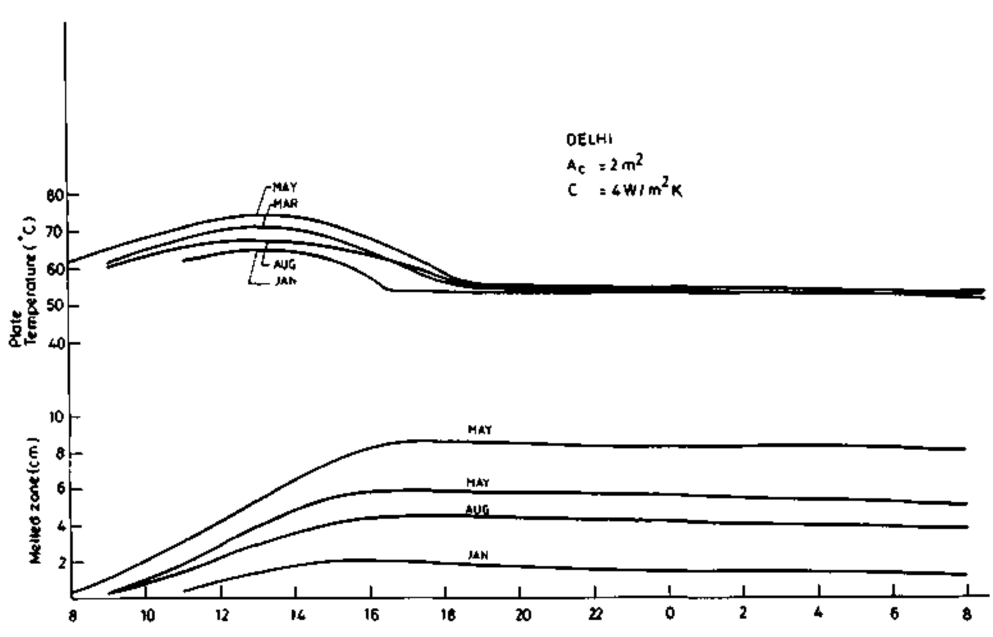

Fig. 4. Hourly variation of melted zone and stagnation temperature with cover during discharging mode.

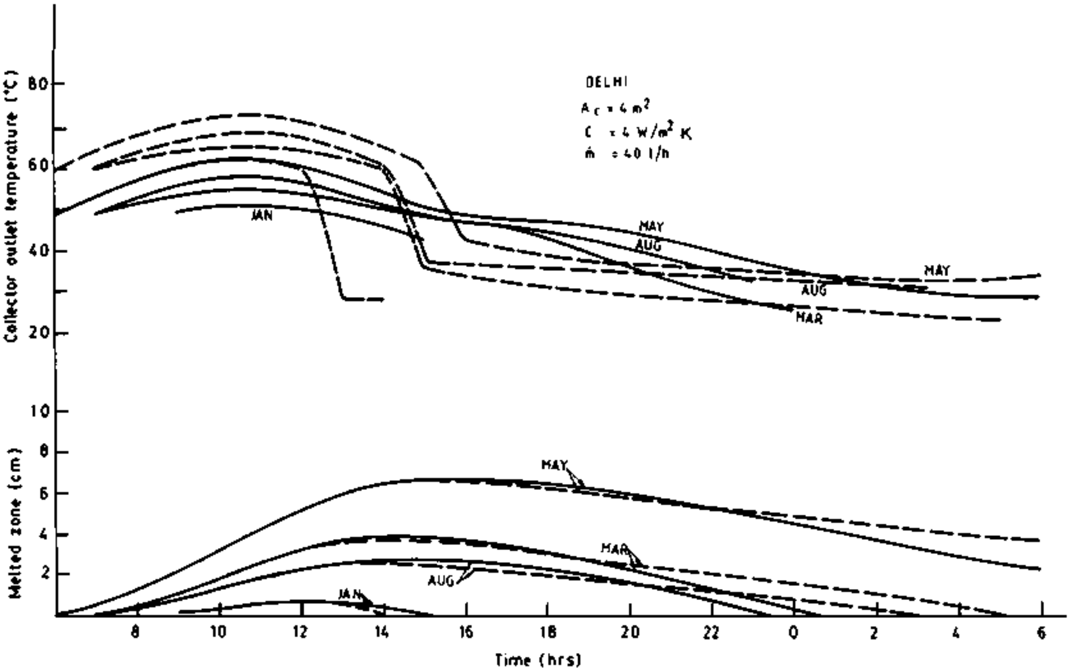

Fig. 5. Hourly variation of melted zone and collector outlet temperature with cover during discharging mode.

Since in practice the water is withdrawn from the system during definite time intervals, calculations have also been performed for the daily demand pattern. These results are shown in Figs. 6 and 7 for infinite and finite conductance of the air gap and for the case of night insulation respectively.

Table 3 shows the variation of efficiencies for four months of the year, characterizing the typical seasonal variations. One of the most striking results in the table is that the thermal efficiency does not show any appreciable seasonal variations, a result that is in contrast to a separate collector and a separate storage system.

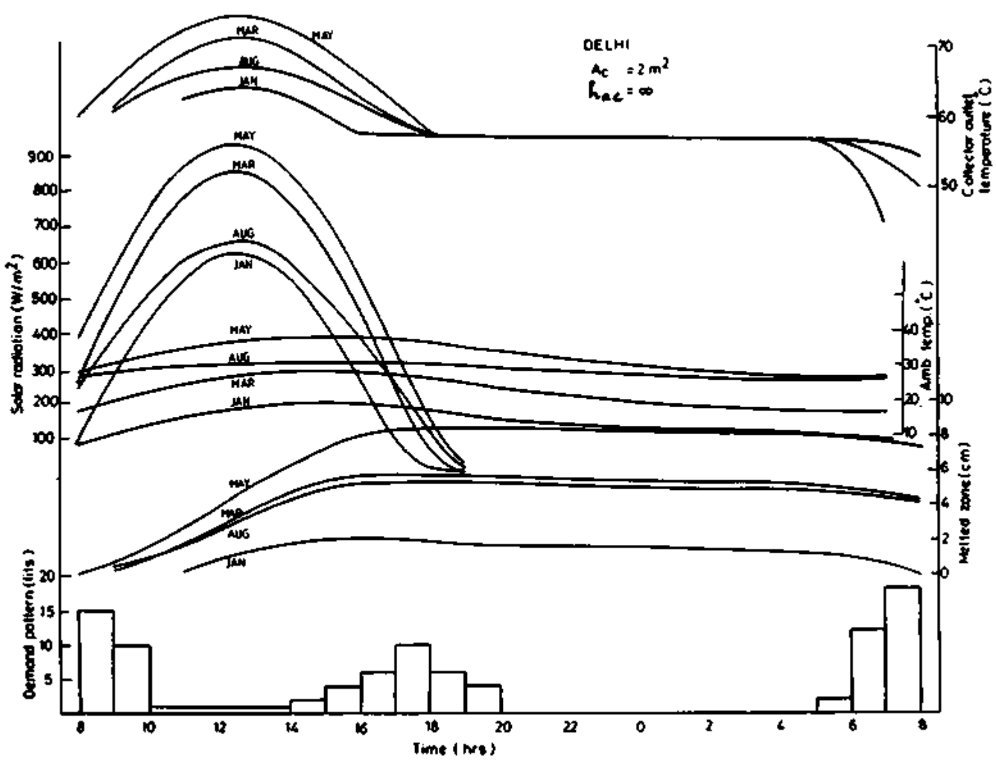

Fig. 6. Hourly variation of melted zone, outlet temperature, demand pattern, solar radiation, and ambient temperature with cover during discharging mode.

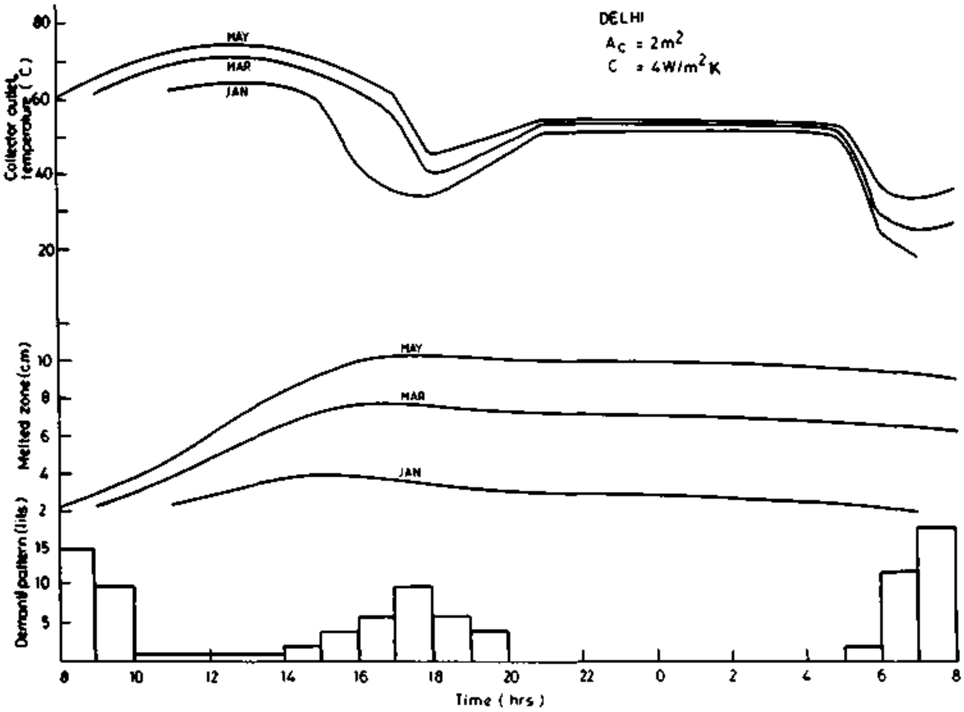

Fig. 7. Hourly variation of melted zone, outlet temperature, and demand pattern with cover during discharging mode.

The table also shows that the efficiency of the system remains nearly constant with the withdrawal rate of useful energy. Therefore, this result clearly supports the conclusion that the employment of PCM as a storage media may be more useful for a solar system in which collector and storage are combined in one unit. On the other hand, for a sensible storage such a system (i.e., a collector cum storage system) yields lower efficiencies and also shows more fluctuations in efficiency and the fluid temperature with the fluctuations in the climatic conditions.

4. CONCLUSIONS

This chapter presents the analysis of a PCM collector cum storage system resulting in the HWB type equations that can be utilized to characterize such systems. The numerical calculations performed clearly show that a PCM collector cum storage system has definite advantages over a system that has a separate collection and a separate storage unit. The system design needs more work to reduce the air gap that results because of the freezing of phase change material in the discharging mode.

NOMENCLATURE

- A_c absorber area of the flat-plate collector (m^2)
- A_s area of the melted zone in contact with the side (m^2)
- C_b bond conductance (W/m^2 K)
- c_p specific heat capacity of the collector fluid (J/kg K)
- d diameter of the tube (m)

Table 3. Efficiency and useful energy calculated for the PCM collector cum storage system during various months of the year†

	Mass flow rate 15 L/h Area of collector 2.0 m^2		Mass flow rate 40 L/h Area of collector 4.0 m^2	
	Mean daily collected energy (kwh/m^2 d)	Efficiency (%)	Mean daily collected energy (kwh/m^2 d)	Efficiency (%)
January	1.66	46.6	1.52	48.5
March	2.57	47.0	2.67	48.9
May	3.36	47.9	3.49	49.8
August	2.35	47.9	2.44	49.8

† Heat loss coefficient, U_{t1} = 6.5 W/m^2 K (charging period). Discharging period, U_{t2} = 0.5 W/m^2 K; h_{ac} = infinite.

d_i internal diameter of the tube (m)
H latent heat of fusion of phase change material (J/kg)
h_a heat loss coefficient from the bottom of the flat-plate collector to the ambient (W/m² K)
h_{ac} air conductance between the absorber and the solid PCM during the discharging mode (W/m² K)
h_c heat transfer coefficient between the absorber and the liquid PCM (W/m² K)
h_{fi} heat transfer coefficient between the fluid and the tube wall (W/m² K)
h_l heat transfer coefficient from the melted zone of the PCM to the solidified zone in the discharging mode (W/m² K)
h_s heat transfer coefficient at the liquid–solid interface (W/m² K)
k thermal conductivity of the absorber (W/m² K)
k_s thermal conductivity of the solid PCM (W/m² K)
L total thickness of the PCM (m)
l length of the tube of the flat-plate collector (m)
m flow rate of the fluid (kg/s)
n number of harmonics (dimensionless)
n_c total number of tubes (dimensionless)
q_u useful heating power (W/m²)
T temperature of the absorber plate between the two fluid tubes (°C)
T_a ambient temperature (°C)
T_{an} nth harmonic in the Fourier series expansion of the ambient temperature (°C)
T_{a0} average value of the ambient temperature (°C)
T_b temperature of the pond (°C)
T_{fi} temperature of the collector fluid at the inlet of the flat-plate collector (°C)
T_{fo} temperature of the collector fluid at the outlet of the flat-plate collector (°C)
T_m melting temperature of the PCM (°C)
T_s temperature of the solid PCM (°C)
T_{si} temperature of the solid PCM (°C)
t time (s)
U_b overall heat loss coefficient from the side to the ambient (W/m² K)
U_{t1} overall heat loss coefficient from the top to the ambient in the charging mode (W/m² K)
U_{t2} heat loss coefficient from the top to the ambient in the discharging mode (W/m² K)
W fin width (m)
X_m thickness of the melted zone during the charging mode (m)
Y_{m1} thickness of the solidified zone during the discharging mode (m)
x, y, z space coordinate (m)

Greek

δ thickness of the absorber (m)
ρ_l density of the liquid PCM (kg/m³)
ρ_s density of the solid PCM (kg/m³)
η collector efficiency (dimensionless)
ω frequency corresponding to 24 hours periodicity, $2\pi/24$ (h⁻¹)

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