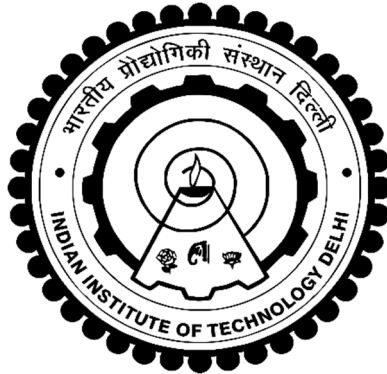


**MECHANICS OF CABLES AND
CABLE-SUSPENDED STRUCTURES**

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**MECHANICS OF CABLES AND
CABLE-SUSPENDED STRUCTURES**

by

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Department of Civil Engineering

Submitted

in the fulfilment of the requirements of the degree of Doctor of Philosophy

to the



Indian Institute of Technology Delhi

May 2018

CERTIFICATE

This is to certify that the Thesis entitled, “**Mechanics of Cables and Cable-Suspended Structures**” being submitted by **Pankaj Kumar** to the **Indian Institute of Technology Delhi**, for the award of degree of **Doctor of Philosophy** is a bonafide record of research work carried out by him under our supervision and guidance. The Thesis, in our opinion, has reached the requisite standard, fulfilling the requirements for the award of degree of Doctor of Philosophy.

The research report and results presented in this Thesis have not been submitted, in part or full, to any University or Institute for the award of any degree or diploma.

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(Pankaj Kumar)

ABSTRACT

Though the research on sagging elastic cables started in eighteenth century, new theories are being proposed even in the present decades. Cables lack unique passive natural state. Their deformed equilibrium state under self-weight is generally employed as reference state. The equations of motion are stated in terms of the additional dynamic forces and the displacements from this reference state. Alternatively, the intrinsic kinematical description is based upon the position vector as a function of the Lagrangian coordinates along the cable length and the assumed constitutive equation relates the local axial tension with the local axial stretch. In addition to physical and geometrical nonlinearities, the presence of self-weight constitutes another source of nonlinearity. The tangent stiffness of inherently nonlinear cables is generally determined as the sum of tangent elastic and geometric stiffness matrices. In contrast, some researchers have preferred to obtain the tangent stiffness matrix by inverting the tangent flexibility matrix.

The following research gaps are identified in the available theories of sagging elastic cables: Sagging elastic cables lack the unique passive natural reference state defined by the coordinates of any point in reference to an external Cartesian coordinate system. Sagging cables exhibit nonlinear response even when all the above sources of nonlinearities are absent. Theories of sagging cables capable of handling the lack of their unique natural reference state and this peculiar nonlinearity are unavailable. The general objective of the present Thesis is to develop the required static and dynamical theories of single sagging cables and cable structures.

First of all, adopting Benipal's approach, a theory of weightless sagging elastic planar cables supporting point forces and concentrated masses undergoing small elastic

displacements is proposed. Natural configuration-defining coordinates and small elastic displacements are shown to be respectively zero and first order homogeneous functions of the loads. Unique natural state corresponding to each equilibrium state is obtained by proportional unloading and is used as the reference state for defining the nodal elastic displacements. New configurational complementary energy is defined and a Castigliano-type theorem for the same is derived. New principles of contra-gradience are proposed. Incremental and rate-type constitutive equations are formulated using tangent configurational and tangent elastic flexibility matrices. Thereafter, new third order differential equation of motion is formulated in terms of loading rates and deformed state nodal coordinates as primary kinematical variables.

A particular two-node four-DOF single sagging cable fixed at both ends is considered for their nonlinear static and dynamic analysis. Subharmonic resonances, beating phenomenon and jump phenomenon are predicted. Inextensible cables are modelled as cables with extremely high axial elastic stiffness. Symmetric elastic vibration mode with smaller total amplitudes introduces higher tensile forces in the cable in contrast to their anti-symmetric configurational vibration mode with greater amplitudes.

The proposed theory is compared with the existing theory of weightless sagging cables regarding both qualitatively and quantitatively. It has been found to be equivalent to, even better in some respects than, the existing theory. This theory is validated with available experimental data.

Using the above theory of sagging elastic cables, a new theory of cable-suspended elastic beams undergoing small elastic displacements is developed. A Beam on Elasto-Flexible Support (BEFS) Model is proposed for simulating the cable-beam interaction.

An iterative procedure for establishing the dead-load configuration is also developed. In contrast to sagging cables, the cable-suspended beams do possess unique natural passive reference state and exhibit nonlinear response even under general proportional loading. Incremental and rate-type constitutive equations as well as third order differential equations of motion are established in terms of vertical nodal beam displacements. The tangent stiffness matrix is obtained by adding the tangent stiffness matrices of the suspension cables and the beam. The proposed theory has been used to determine the static and dynamic response of linear and nonlinear elastic beams. Anti-symmetric and symmetric modes of vibration of beams correspond respectively to the configurational and the elastic response of suspension cables. A new cross-over criterion based upon diagonal stiffness matrix is proposed in contrast to the well-known equal modal frequency criterion.

For the determination of seismic response of these cable structures, rate of ground acceleration input is required apart from the available ground acceleration data itself. Horizontal and vertical seismic excitations introduce respectively predominantly configurational and elastic response of single sagging cables. Similarly, symmetric flexural and torsional vibrations of the beam along with elastic displacements of the suspension cables are caused by the vertical seismic excitations. In all these cases, the dominant response frequencies match some of the dominant excitation frequencies or natural frequencies of the structure. The linearization scheme generally used for nonlinear elastic structures is based upon second order differential equation of motion. It is found to be incapable of predicting subharmonic resonances, vibration response of inextensible cables, beating phenomenon, configurational response of cables, seismic response, etc.

Thus, a new theory of sagging elastic cables is proposed and compared with existing displacement theory. The proposed theory is also applied to cable-suspended beams for investigating their static and dynamic response. Fertile areas of further research are identified.

सार

हालांकि अठारहवीं शताब्दी में लोचदार केबलों को शुरू करने पर शोध शुरू हुआ, लेकिन मौजूदा दशकों में भी नए सिद्धांतों का प्रस्ताव दिया जा रहा है। केबल्स में अद्वितीय निष्क्रिय प्राकृतिक अवस्था की कमी है। स्व-वजन के तहत उनके विकृत समतोल स्थिति को आम तौर पर संदर्भ स्थिति के रूप में कार्यरत किया जाता है। गति के समीकरण अतिरिक्त गतिशील ताकतों और इस संदर्भ स्थिति से विस्थापन के संदर्भ में वर्णित हैं। वैकल्पिक रूप से, आंतरिक किनेमेटिकल विवरण केबल वेक्टर के साथ लैग्रेंगियन निर्देशांक के एक समारोह के रूप में स्थिति वेक्टर पर आधारित होता है और अनुमानित संवैधानिक समीकरण स्थानीय अक्षीय खिंचाव के साथ स्थानीय अक्षीय तनाव से संबंधित है। भौतिक और ज्यामितीय गैर रेखीय के अलावा, आत्म वजन की उपस्थिति रेखीय के एक और स्रोत का गठन किया। स्वाभाविक रूप से रेखीय केबल्स की स्पज्या कठोरता आमतौर पर स्पज्या लोचदार और ज्यामितीय कठोरता *matrices* के योग के रूप में निर्धारित किया जाता है। इसके विपरीत, कुछ शोधकर्ताओं ने स्पज्या लचीलापन मैट्रिक्स को बदलकर स्पज्या कठोरता मैट्रिक्स प्राप्त करना पसंद किया है।

निम्न शोध अंतराल को लोचदार केबल्स को उपलब्ध कराने के उपलब्ध सिद्धांतों में पहचाना जाता है: बाहरी कार्टेसियन समन्वय प्रणाली के संदर्भ में किसी भी बिंदु के निर्देशांक द्वारा परिभाषित अद्वितीय निष्क्रिय प्राकृतिक संदर्भ स्थिति को कम करने वाले लोचदार केबल्स में कमी होती है। सागिंग केबल्स रेखीय प्रतिक्रिया प्रदर्शित करते हैं, भले ही गैर रेखीय के सभी उपरोक्त स्रोत अनुपस्थित हैं। अपने अद्वितीय प्राकृतिक संदर्भ स्थिति की कमी को संभालने में सक्षम सगेबिंग केबल्स की सिद्धांत और यह असाधारण गैर-रेखाचित्र अनुपलब्ध है। वर्तमान थीसिस का सामान्य उद्देश्य एकल सगाई केबल्स और केबल संरचनाओं के आवश्यक स्थिर और गतिशील सिद्धांतों को विकसित करना है।

सबसे पहले, बेनिपाल के दृष्टिकोण को अपनाने, वजन रहित सागिंग लोचदार प्लानर केबल्स का एक सिद्धांत बिंदु बलों और छोटे लोचदार विस्थापन से गुजरने वाले केंद्रित लोगों का समर्थन करता है। प्राकृतिक विन्यास-परिभाषित निर्देशांक और छोटे लोचदार विस्थापन क्रमशः शून्य होते हैं और लोड के पहले क्रमबद्ध कार्यों को क्रमबद्ध करते हैं। प्रत्येक समतोल स्थिति के अनुरूप अद्वितीय प्राकृतिक अवस्था

आनुपातिक अनलोडिंग द्वारा प्राप्त की जाती है और नोडल लोचदार विस्थापन को परिभाषित करने के लिए संदर्भ स्थिति के रूप में उपयोग की जाती है। नई विन्यास पूरक पूरक ऊर्जा परिभाषित की जाती है और इसके लिए एक कास्टिग्लियानो-प्रकार प्रमेय व्युत्पन्न होता है। संकुचन के नए सिद्धांत प्रस्तावित हैं। वृद्धिशील और दर-प्रकार संवैधानिक समीकरण स्पज्या कॉन्फ़िगरेशन और स्पज्या लोचदार लचीलापन मैट्रिस का उपयोग करके तैयार किए जाते हैं। इसके बाद, लोडिंग दरों और विकृत स्थिति नोडल निर्देशांक प्राथमिक किनेमेटिकल चर के रूप में गति के नए तीसरे क्रम अंतर समीकरण तैयार किए जाते हैं।

दोनों सिरों पर तय एक विशेष दो-नोड चार-डीओएफ सिंगल सैगिंग केबल को उनके रेखीय स्थिर और गतिशील विश्लेषण के लिए माना जाता है। Subharmonic अनुनाद, घटना को मारने और घटना कूदने की भविष्यवाणी की जाती है। अतुलनीय केबलों को अत्यधिक उच्च अक्षीय लोचदार कठोरता वाले केबल्स के रूप में मॉडलिंग किया जाता है। छोटे कुल आयामों के साथ सममित लोचदार कंपन मोड केबल में उच्च तन्यता बलों को उनके एंटी-सममित कॉन्फ़िगरेशन कंपन मोड के विपरीत अधिक आयामों के साथ प्रस्तुत करता है।

प्रस्तावित सिद्धांत की गणना गुणात्मक और मात्रात्मक दोनों के संबंध में भार रहित sagging केबल्स के मौजूदा सिद्धांत के साथ की जाती है। मौजूदा सिद्धांत की तुलना में यह कुछ मामलों में भी बेहतर है। यह सिद्धांत उपलब्ध प्रयोगात्मक डेटा के साथ मान्य है।

लोचदार केबल्स को कम करने के उपर्युक्त सिद्धांत का उपयोग करते हुए, छोटे लोचदार विस्थापन से गुजरने वाले केबल-निलंबित लोचदार बीम का एक नया सिद्धांत विकसित किया गया है। केबल-बीम इंटरैक्शन को अनुकरण करने के लिए एलिस्टो-फ्लेक्सिबल सपोर्ट (बीईएफएस) मॉडल पर एक बीम प्रस्तावित है। मृत लोड विन्यास स्थापित करने के लिए एक पुनरावृत्ति प्रक्रिया भी विकसित की गई है। केबलों को सील करने के विपरीत, केबल-निलंबित बीम में अद्वितीय प्राकृतिक निष्क्रिय संदर्भ स्थिति होता है और सामान्य आनुपातिक लोडिंग के तहत भी गैर-लाइनर प्रतिक्रिया प्रदर्शित करता है। गतिशील नोडल बीम विस्थापन के संदर्भ में गतिशील और दर-प्रकार संवैधानिक समीकरणों के साथ-साथ गति के तीसरे क्रम अंतर समीकरण स्थापित किए जाते हैं। स्पज्या कठोरता मैट्रिक्स को निलंबन केबल्स और बीम के स्पज्या

कठोरता मैट्रिस जोड़कर प्राप्त किया जाता है। प्रस्तावित सिद्धांत का उपयोग रैखिक और रेखीय लोचदार बीम की स्थिर और गतिशील प्रतिक्रिया निर्धारित करने के लिए किया गया है। बीम के कंपन के एंटी-सममित और सममित मोड क्रमशः कॉन्फ़िगरेशनल और निलंबन केबल्स की लोचदार प्रतिक्रिया से मेल खाते हैं। विकर्ण कठोरता मैट्रिक्स के आधार पर एक नया क्रॉस-ओवर मानदंड प्रस्तावित बराबर मोडल आवृत्ति मानदंड के विपरीत प्रस्तावित किया गया है।

इन केबल संरचनाओं की भूकंपीय प्रतिक्रिया के निर्धारण के लिए, उपलब्ध जमीन त्वरण डेटा के अलावा जमीन त्वरण इनपुट की दर आवश्यक है। क्षैतिज और ऊर्ध्वाधर भूकंपीय उत्तेजना क्रमशः मुख्य रूप से कॉन्फ़िगरेशनल और एकल सगाई केबल्स की लोचदार प्रतिक्रिया प्रस्तुत करते हैं। इसी तरह, निलंबन केबल्स के लोचदार विस्थापन के साथ बीम के सममित लचीला और टोरसोनियल कंपन ऊर्ध्वाधर भूकंपीय उत्तेजनाओं के कारण होते हैं। इन सभी मामलों में, प्रमुख प्रतिक्रिया आवृत्तियों में कुछ प्रमुख उत्तेजना आवृत्तियों या संरचना की प्राकृतिक आवृत्तियों से मेल खाते हैं। आमतौर पर रेखीय लोचदार संरचनाओं के लिए उपयोग की जाने वाली रैखिकरण योजना गति के दूसरे क्रम अंतर समीकरण पर आधारित है। यह **subharmonic** अनुनाद, अनुमानित केबलों की कंपन प्रतिक्रिया, घटना को मारने, केबलों की विन्यास प्रतिक्रिया, भूकंपीय प्रतिक्रिया, आदि की भविष्यवाणी करने में असमर्थ पाया जाता है।

इस प्रकार, लोचदार केबल्स को सगाई करने का एक नया सिद्धांत प्रस्तावित और मौजूदा विस्थापन सिद्धांत की तुलना में किया जाता है। प्रस्तावित सिद्धांत भी उनके स्थैतिक और गतिशील प्रतिक्रिया की जांच के लिए केबल-निलंबित बीम पर लागू होता है। आगे के शोध के उपजाऊ क्षेत्रों की पहचान की जाती है।

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LIST OF NOTATIONS

A_0	Cross-sectional area of the cable
a_0, a_1	Damping coefficients
B_{ij}	Tangent transformation matrix
C_{ij}	Constant damping matrix
D_{ij}	Tangent configurational flexibility matrix
E_c	Modulus of elasticity of the cable
E_d	Modulus of elasticity of the deck
e	Eccentricity
F_{ij}, \dot{F}_{ij}	Applied nodal load and load-rate vectors
F_0	Applied constant sustained nodal load vector
F_d	Force balanced by deck
F_L	Applied peak harmonic nodal load vector
f_{ij}	Tangent elastic flexibility matrix of the cable
f_d	Tangent elastic flexibility matrix of the deck
g	Acceleration due to gravity
G	Tangent elasto-configurational stiffness matrix of the cable

H	Vertical distance between the fixed supports of sagging cable
G	Modulus of rigidity of deck
K_{ij}	Tangent elasto-configurational stiffness matrix of the cable
k_c	Tangent elasto-configurational stiffness matrix of the cable
k_d	Tangent stiffness matrix of the deck
k_e	Tangent elastic stiffness matrix of the cable
k_g	Tangent geometrical stiffness matrix of the cable
k_{eg}	Tangent elasto-geometrical stiffness matrix of the cable
L	Horizontal distance between the fixed supports of sagging cable
M_{ij}	Mass matrix
N_{ij}	Tangent elasto-configurational flexibility matrix of the cable
P_i, \dot{P}_i	Resistive nodal load and load-rate vectors
p_i	Eigenvectors
r	Bilinearity ratio of cracked concrete beam
S_r	Length of the r^{th} cable segment
T_r	Tensile force in the r^{th} cable segment
u_i, \dot{u}_i	Nodal elastic displacement and velocity of cable node

u_{ei}	Nodal elastic displacement from equilibrium configuration of cable
v_i, \dot{v}_i	Nodal elastic displacement and velocity of beam node
v_{ei}	Nodal elastic displacement from equilibrium state of beam
x_i, \dot{x}_i	Nodal coordinates and nodal velocities in the corresponding undeformed state of the cable
x_{i0}	Initial nodal coordinates in the undeformed state of the cable
x_{ei}	Nodal configurational increment from equilibrium state of the cable
$y_i, \dot{y}_i, \ddot{y}_i$	Nodal coordinates, velocities and acceleration in the deformed state of the cable
y_{i0}	Initial nodal coordinates in the deformed state of the cable
z_i	Nodal displacements from equilibrium state of cable
Δ_r	Axial elastic displacement in the r^{th} cable segment
ξ	Modal damping ratio
Π_c	Configurational complementary potential energy of an inextensible cable
ω_{ni}	Modal frequencies of vibration
Ω	Total complementary energy of the elastic cable
Ω_c	Configurational complementary energy of the elastic cable
Ω_e	Elastic complementary energy of the elastic cable

λ_i Eigenvalues