ON ESTIMATION OF MARKOV PROCESSES CORRUPTED BY WHITE GAUSSIAN NOISE

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CERTIFICATE

This is to certify that the dissertation, "On Estimation of Markov Processes Corrupted by White Gaussian Noise", which is being submitted by Arogyaswami Joseph Paulraj for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology, New Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

This dissertation has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this dissertation have not been submitted to any other University or Institute for the award of any degree or diploma.

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SYMBOLS

$\mathcal{A}$	σ-field on Ω
$\mathcal{A}_{X_{ts}}$	σ-field generated by $\{X_u,\ t \leqslant u \leqslant s\}$
$\mathcal{P}$	probability measure on Ω
$\mathcal{A}_t$	sub σ-field of $\mathcal{A}$
I_A	indicator function for set A
$\mathcal{L}$	differential operator
R	real line
$W(\omega, t)$	Wiener process
$\sigma\{X_u,\ 0 \leqslant u \leqslant t\}$	σ-field generated by $\{X_u,\ 0 \leqslant u \leqslant t\}$
$E[.\|.]$	conditional expectation
$E_*[.]$	expectation with respect to measure $\mathcal{P}_*$
$p(.,.)$	apriori density
$p(.,.\|.,.)$	transition density
π_0, π_1	apriori probabilities
$<<$	absolute continuity relation
$\perp\!\!\!\perp$	independence relation
$\sigma\{.\} \perp\!\!\!\perp \mathcal{A}\|X_t$	$\sigma\{.\}$ is conditionally independent of $\mathcal{A}$ given X_t
$t \wedge s$	$\min(t,s)$
$t \vee s$	$\max(t,s)$
ρ	Lebesgue measure
$\|\|$	end of proof symbol

ABSTRACT

This dissertation is a study of the problem of continuous time estimation of Markov signals corrupted by white Gaussian noise. Specifically, it is aimed at obtaining a unified solution to the various sub-classes of estimation problems such as filtering, smoothing, prediction etc. This result for a general Markov process, which we term as the generalized solution, is applied to a specific Markov model - the diffusion process, to arrive at a generalized stochastic differential equation. This equation is a general result and is shown to yield solutions to a variety of estimation problems. The extensions of the above results to the case of uncertain observations is also considered in this dissertation. The basic results are obtained by appropriate extensions of certain well known theorems in the literature of estimation theory. The approach taken in this dissertation provides a framework of sufficient generality to encompass a much wider class of problems.

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