

ON ESTIMATION OF MARKOV PROCESSES CORRUPTED BY WHITE

GAUSSIAN NOISE

AROGYASWAMI JOSEPH PAULRAJ

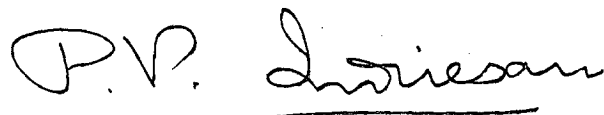
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CERTIFICATE

This is to certify that the dissertation, "On Estimation of Markov Processes Corrupted by White Gaussian Noise", which is being submitted by Arogyaswami Joseph Paulraj for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology, New Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

This dissertation has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this dissertation have not been submitted to any other University or Institute for the award of any degree or diploma.



(P.V. Indiresan)

Department of Electrical Engineering  
Indian Institute of Technology  
New Delhi - 29

INDIA

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*Arogyaswami Paulraj*

## SYMBOLS

$\mathcal{a}$	$\sigma$ -field on $\Omega$
$\mathcal{a}_{X_{ts}}$	$\sigma$ -field generated by $\{X_u, t \leq u \leq s\}$
$\mathcal{P}$	probability measure on $\Omega$
$\mathcal{a}_t$	sub $\sigma$ -field of $\mathcal{a}$
$I_A$	indicator function for set $A$
$\mathcal{L}$	differential operator
$\mathbb{R}$	real line
$W(\omega, t)$	Wiener process
$\sigma\{X_u, 0 \leq u \leq t\}$	$\sigma$ -field generated by $\{X_u, 0 \leq u \leq t\}$
$E[. .]$	conditional expectation
$E_*[.]$	expectation with respect to measure $\mathcal{P}_*$
$p(.,.)$	a priori density
$p(.,. .,.)$	transition density
$\pi_0, \pi_1$	a priori probabilities
$\ll$	absolute continuity relation
$\parallel$	independence relation
$\sigma\{.\} \parallel \mathcal{a} X_t$	$\sigma\{.\}$ is conditionally independent of $\mathcal{a}$ given $X_t$
$t \wedge s$	$\min(t, s)$
$t \vee s$	$\max(t, s)$
$\rho$	Lebesgue measure
$\parallel$	end of proof symbol

## ABSTRACT

This dissertation is a study of the problem of continuous time estimation of Markov signals corrupted by white Gaussian noise. Specifically, it is aimed at obtaining a unified solution to the various sub-classes of estimation problems such as filtering, smoothing, prediction etc. This result for a general Markov process, which we term as the generalized solution, is applied to a specific Markov model - the diffusion process, to arrive at a generalized stochastic differential equation. This equation is a general result and is shown to yield solutions to a variety of estimation problems. The extensions of the above results to the case of uncertain observations is also considered in this dissertation. The basic results are obtained by appropriate extensions of certain well known theorems in the literature of estimation theory. The approach taken in this dissertation provides a framework of sufficient generality to encompass a much wider class of problems.

## CONTENTS

<u>CHAPTER</u>		<u>Page</u>
	ABSTRACT	1
I	INTRODUCTION AND PRELIMINARIES	
	Preface	2
	A History of the Problem	4
	B New Results	8
	C A Note on Mathematical Techniques	
	1. General Theory and Notation	10
	2. Stochastic Integrals and Stochastic Calculus	13
II	ESTIMATION OF RANDOM PROCESSES CORRUPTED BY WHITE GAUSSIAN NOISE	
	Preface	16
	A Preliminaries	
	1. Signal Model	17
	2. Estimation Problem	20
	B Absolute Continuity and RND of Certain Measures with respect to the Wiener Measure	21
	C Some Results for the Generalized Con- ditional Expectation of Random Signals Corrupted by White Gaussian Noise	24
III	ESTIMATION OF MARKOV PROCESSES CORRUPTED BY WHITE GAUSSIAN NOISE	
	Preface	30
	A Some Results for Evolution of Certain Conditional RNDs	30

<u>CHAPTER</u>		<u>Page</u>
	B Main Result : A Theorem for Generalized Estimation of Markov Processes Corrupted by White Gaussian Noise	39
	C Some Results for Conditional Expectations of Markov Processes	44
IV	A GENERAL STOCHASTIC DIFFERENTIAL EQUATION SATISFIED BY THE CONDITIONAL DENSITY OF DIFFUSION PROCESSES CORRUPTED BY WHITE GAUSSIAN NOISE	
	Preface	47
	A Diffusion Process Signal Model	49
	B Stochastic Differential Equations Satisfied by Certain Conditional RNDs	52
	C Main Result : A General Stochastic Differential Equation Satisfied by the Conditional Density of Diffusion Processes	62
V	SOME SPECIAL CASES FOR ESTIMATION OF DIFFUSION PROCESSES	
	Preface	71
	A Filtering Problem	74
	B Smoothing Problem	
	1. Fixed Interval Smoothing	76
	2. Fixed Point Smoothing	78
	3. Fixed Lag Smoothing	78
	C Prediction Problem	
	1. Fixed Data Prediction	83
	2. Fixed Point Prediction	84

<u>CHAPTER</u>	<u>Page</u>
3. Fixed Lead Prediction	86
D Retrodiction Problem	
1. Fixed Data Retrodiction	90
2. Fixed Point Retrodiction	91
3. Fixed Lag Retrodiction	93
VI ESTIMATION OF MARKOV PROCESSES WITH UNCER- TAINTY IN THE OBSERVATIONS AND CONCLUDING REMARKS	
Preface	95
A Preliminaries and General Results	
1. Signal Model	96
2. A General Result for Estimation of Markov Signals under Uncertain Observations	99
B Applications to Diffusion Processes	101
C Concluding Remarks and Suggestions for Further Research	106
REFERENCES	109