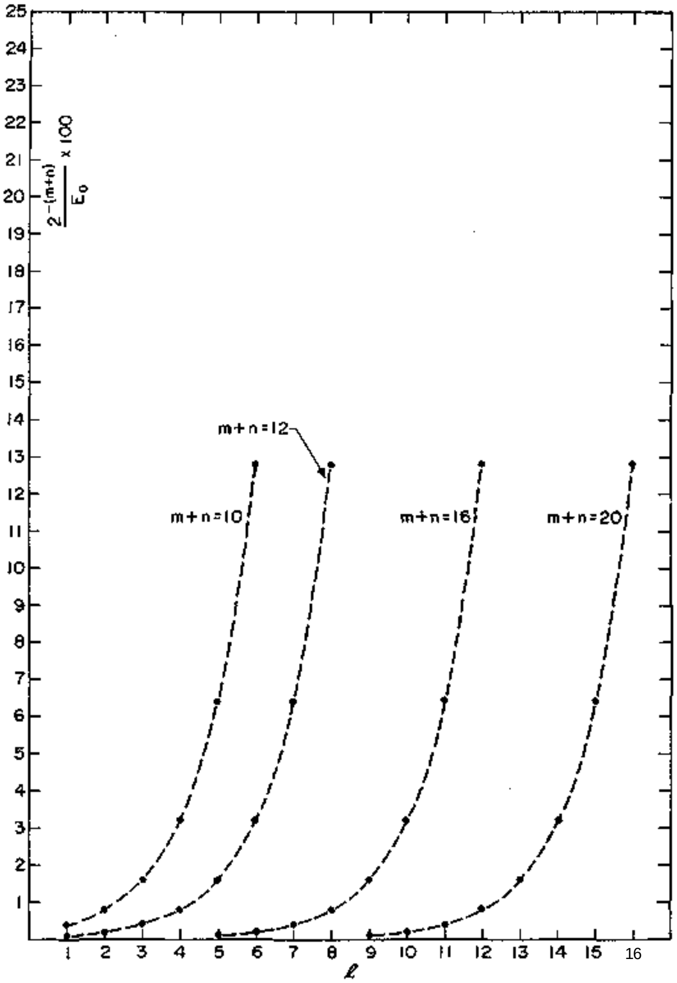

Fig. 1. Percentage difference in the two expressions for the mean-squared error.

Substituting for $^{-2}$, the error in the output of the filter due to this source of product quantization error becomes

$$E[|v_i|^2] = \frac{E_0}{3}\left[E_0 + 2^{-(m+n)}\right] \sum_{k=0}^{i} |h_k|^2. \quad (10)$$

Thus, the difference in the model for product quantization affects the output error as seen in (10). This result is general enough to consider different quantization levels for the input, coefficients and products of a digital filter. For example, if the input is rounded to m binary bits, the coefficients to n bits, and the products (hence, the output) to 1 bits, the mean square of the error in the output due to the quantization of the products is given by (10). The usual model gives the same value for any quantization level that is used for the input and coefficients.

Fig. 1 shows a plot of the percentage difference E between the two expressions for the mean squared error due to product quantization for various values of $m+n$.

$$E = \frac{(E_0/3)(E_0 + 2^{-(m+n)}) - (E_0^2/3)}{(E_0^2/3)} \times 100 = \frac{2^{-(m+n)}}{E_0}. \quad (11)$$

For a fixed quantization level, say rounding to 1 binary bits, the percentage difference becomes smaller as $m+n$ increases. This is expected since as $m+n$ increases, the product to be quantized approaches a continuous variable; hence, the difference between the two expressions becomes smaller. For similar reasons, when $m+n$ is fixed the percentage difference increases as 1 increases. It should also be noted that when $m=n=1$, the percentage difference decreases as the quantization level increases.

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An Active Feedback Block and its Applications

Abstract

The active feedback block considered in this correspondence consists of three identical single-pole amplifiers and a resistive network, and has a positive slope in the gain frequency characteristics over a certain frequency range starting from dc. When the device is cascaded to another single-pole amplifier, the bandwidth and the gain-bandwidth product of the overall system are found to be increased compared to those of a cascaded two-stage amplifier.

High-frequency compensation of amplifiers is a classical subject and standard methods, using inductance and passive feedback techniques, are described in detail in standard textbooks on electronic circuits. In microcircuits, one cannot use an inductance; on the other hand, passive feedback techniques have their limitations with regard to the total gain-bandwidth product (GBW). In this communication, we propose a different approach to compensation, using the so-called "active feedback block" (AFB), which is particularly suitable for microcircuit construction and application. The schematic diagram of the AFB is shown in Fig. 1(A), where amplifiers 1, 2, and 3 are identical, each having a gain

$$A_i(s) = -A_0\omega_0/(s+\omega_0), \quad A_0 > 0, \quad (1)$$

and Po represents the transfer function of a

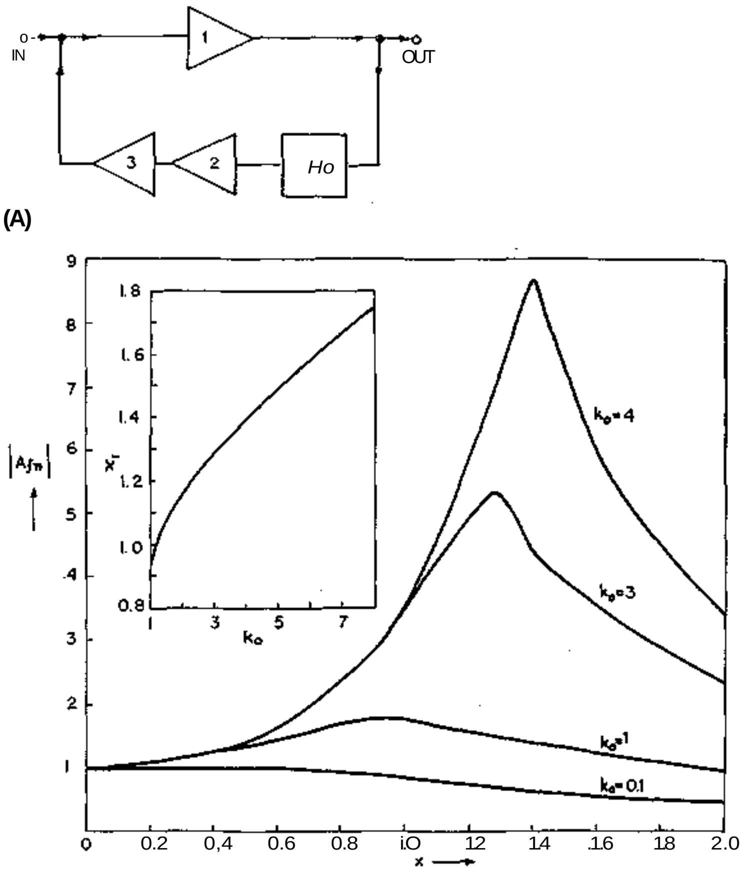

Fig. 1. (A) Schematic diagram of the active feedback block. (B) Variation of x1 with ko and the frequency response curves of the AFB.

resistive network in the feedback path. The overall gain of the AFB is given by

$A_f(s) = -A_n wo(S + \sim o)'/$

$\cdot[(« + WO)' + knwo3], \quad (2)$

where $k0=/\&A_o^3$. Normalizing the gain with respect to its dc value $A,(0) = A_o/(l+ko)$, and the complex frequency s with respect to wo, we get

$ii_{fn}(s_n)A\Delta_J(\sim)/'A_J(O) = (1+ko)$

$\cdot (s_n + 1)V(s»^3 + 3\sim^2 + 3s_n + I + Go) \quad (3)$

where $s,=s/o\sim$. Equation (3) shows that, depending on k_o, the AFB may be stable or unstable. By applying the Routh-Hurwitz criterion, the condition of stability can be easily shown to be $ko<S$. For &0 = 8, (3) has jo axis poles at $s_n = \& j d\bar{x}$ i.e., the system will oscillate at a frequency d3io.

Applying Bode plot considerations to (3, it appears possible to get a positive slope in the gain frequency sketch in a certain frequency range. In order to investigate this, we let s=jw (sinusoidal excitation) and $x=o/w_o$ in (3) and differentiate the magnitude of the resulting expression with respect to x. The result is, after some algebraic manipulations,

$$\frac{d|A_{fn}(jx)|}{dx} = \frac{(1+k_0)x[(2k_0^2+10k_0-1) - 3(2k_0+1)x^2 - 3x^4 - x^6]}{[(1+k_a-3a;^2)^2 + z^2(3-a;^2)^2]^{3/2}}. \quad (4)$$

It is observed from (4) that at sufficiently high frequencies, the slope will always be negative and that the condition for positive slope is

$$2k_0^2 1 + 10k0 - 1 > 0, \text{ i.e., } k_o > 0.0975. \quad (5)$$

If (5) is satisfied, then the slope will be nonnegative for a certain range of frequencies $0 \le x \le xl$ and will be negative for $x>xl$. We can find $x1$ by equating (4) to zero; the result is

$$x_l = [(a + b)''3 - (a - 6)''^3 - 1^1 1^2$$

where

$$a = k_0(k_0^2 + 24ko + 64)^{1/2}$$

and

$$b = k_0(k_0 + 8). \quad (6)$$

Fig. 1(B) shows the plot of $x1$ versus k_o and also the frequency response for several values of ko.

The rising gain frequency characteristics of the AFB in the range $0 \le x \le xl$ can be used to compensate for the high-frequency falloff of an amplifier. Consider, for illustration, the system shown in Fig. 2(A), which is a cascade of the AFB and an amplifier having the gain given by (1). The overall gain of this system is $A\&)=\&)AL(S)$; normalizing this with respect to the dc value $A_T(O)=A_\sim/(\sim +ko)$, we get the following magnitude squared response for sinusoidal excitation.

$$|A_{Tn}(jx)|^2 = (1+k_0)^2(1+x^2)/$$
$$\cdot[(1+k_0)^2 + 3x^2(1 -21\sim 0) + 3x^4 + x'], \quad (7)$$

From (7), the condition for maximally flat response can be easily shown to be $k_o=0.242$; the frequency response of (7) under this condition is shown in Fig. 2(B), where from the 3-dB bandwidth xh is found to be 0.938. The gain-bandwidth product of compensated amplifier is given by

$$GBW \triangleq A_T(0)x_h\omega_0$$
$$= [x_h/(1 + ko)]Ao'w_o. \quad (8)$$

For the maximally flat case, therefore,

$$GBW \text{-}\sim MF = (0.754do)(\text{-}4aw_o). \quad (9)$$

We thus see that the AFB acts as a GBW booster to the single-stage amplifier $A1$; however, it reduces the bandwidth slightly. If ko=O, i.e., there is no feedback in the AFB, then Fig. 2(A) becomes a cascade of two identical stages, for which xh is known to be 0.644; thus from (S),

$$GBW\ I_{\sim\text{-}\sim tag}e \text{ cascade} = (0.644_Ao)(Ao)(A_0\omega_0). \quad (10)$$

Thus, compared to the two-stage cascade, the compensated amplifier has 1.46 times higher bandwidth and 1.17 times higher GBW.

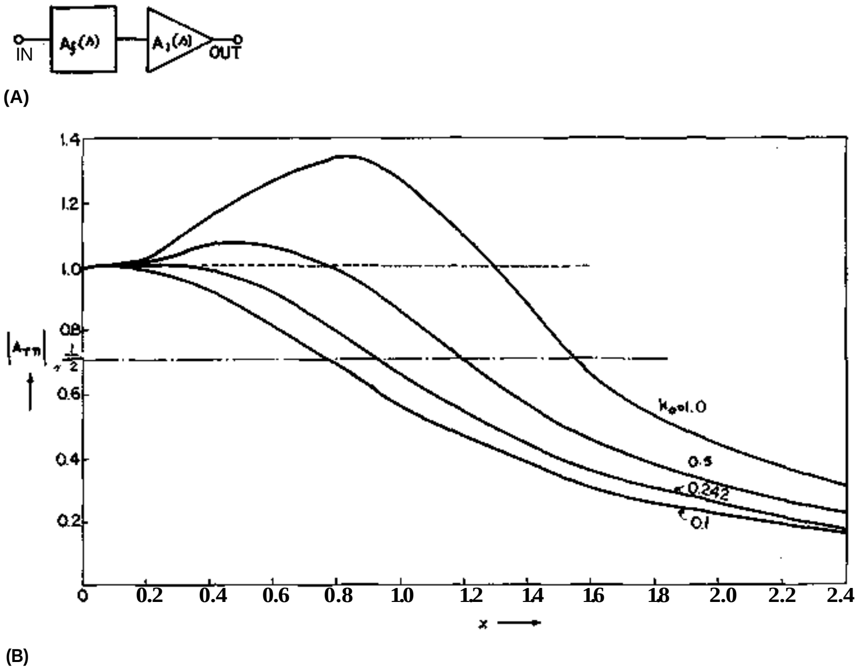

Fig. 2. (A) High-frequency compensation using the AFB. (B) Frequency response curves of the compensated amplifier.

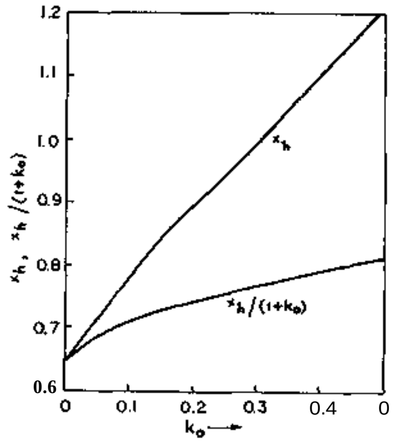

Fig. 3. Bandwidth and GBW of the compensated amplifier as a function of *ko*.

From the frequency response curves shown in Fig. 2(B), it can be seen that for $k_0 > 0.242$ we get peaking, and that if a small amount of peaking can be tolerated, then we can make $x_h > 1$, i.e., the AFB also boosts the bandwidth. The condition for $x_h > 1$ is $|A_{TN}(jx)|^2_{x=1} > 1/2$, which, combined with (7), gives $k_o > 0.321$. The variation of x_h with k_o is shown in Fig. 3.

With $k_0 > 0.242$, there occurs a further improvement in the GBW, as shown by the increase of $x_h/(1+k_o)$ with k_o (Fig. 3).

Further theoretical work on modifications of the scheme for more effective bandwidth and GBW boosting is in progress. Experimental work is also being planned for verifying the theoretical findings. These results will be published in due course.

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